One Risk, Two Debts: The Effects of Rare Disasters on Credit Markets

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Abstract

The COVID-19 pandemic has highlighted the impacts that rare disasters can have on credit markets. We discuss and quantify the asset-pricing implications of disaster risk on the risk-free rate, credit spreads, and their term structures. The findings underscore the heterogeneous effects of disasters on the risk-free and risky debt segments of credit markets. The results reveal that federal and private debt are "two sides of the same coin", call for a closer coordination between these two distinct sectors of the credit market, and shed light on deleveraging issues that likely lie ahead in the post-disaster world.

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Introduction

The COVID-19 pandemic and several policy measures taken to help contain its spread have wreaked economic havoc and disrupted credit markets worldwide. To bolster credit markets, the US Federal Reserve System has taken unprecedented measures. In addition to using conventional monetary policies, it has pledged to purchase an unlimited amount of treasuries and mortgage-backed securities. More notably, it has intervened in the corporate bond market directly, setting up the primary- and secondary-market corporate credit facilities to make outright purchases of corporate bonds and corporate bond exchange-traded funds. These unprecedented policies highlight the Fed's concern about the potential damage that the pandemic might inflict on credit markets.

In this paper, we study the potential impacts that widespread disasters can have on credit markets. To this end, we extend the canonical asset-pricing framework, and study an endowment economy with two types of heterogeneous-belief agents. The endowment economy is subject to risk from rare disasters. Nevertheless, asset markets are not complete because the agents cannot fully hedge against the disaster risk, and, hence, it is possible that some agents will default on their debt. As a consequence, the credit market consists of two segments: risk-free debt and risky debt. Heterogeneous beliefs lead to active trading of both types of debt in the credit market, but agents might default on the risky debt. This distinctive feature of the model thus enables us to characterize the impacts of disaster risk on the risk-free and risky debt markets separately, and to study the potential disruption to the credit markets when disaster materializes. Despite incomplete markets and heterogeneous agents, the model is tractable, and it yields many analytical results.

A key result highlights that an incomplete hedge against disaster risk can account for the positive association between the quantities of risk-free and risky debt. Although we study these types of debt separately, as two distinct sectors of the credit market, they interact with each other closely. In fact, Friedman (1981) surmised that increases in federal borrowing would curtail private borrowing, implying a negative association between the two. Nonetheless, the data cast doubt on this view. Figure 1 plots the quarterly amount of US federal government debt and other types of risky debt (in real terms) from 1966 to 2008, demonstrating a strong positive correlation. Summers (1986) reached a similar conclusion that increases in government debt are actually associated with increases in private debt.

Indeed, our model indicates that federal debt and risky debt are two sides of the same coin. The ratio between the amount of the two types of debt is determined by loss sharing embedded in the risky debt between the creditors and debtors upon default. The results have important policy implications. The roughly $2 trillion economic stimulus package provided by the Coronavirus Aid, Relief, and Economic Security (CARES) Act adds to US Treasury debt, which has been growing for a long time. Deleveraging is anticipated to be a critical political and economic issue in the post-COVID-19 world. Our model shows that public debt and private debt are "twin" problems. Thus, successful deleveraging of either sector is contingent on deleveraging of both sectors, calling for a close coordination between the two sectors.

Disasters can disrupt credit markets, but the matter of whether a disaster will induce default on debt is highly contingent on the endogenous state of the economy at the time. A disaster does not necessarily lead to economy-wide default on debt. The key lies in the market leverage. We show that large belief dispersion between agents, low ex-ante disaster risk, or highly skewed wealth distribution can lead to high market leverage, and exacerbate the credit market disruption once a disaster materializes, resulting in higher loss given default. Moreover, the question of whether disasters result in debt default also affects wealth redistribution, and, hence the price and quantity of the credit in post-disaster credit markets. Default on risky debt effectively works as an imperfect loss-sharing device between debtors and creditors, and leads to lesser change in wealth distribution than otherwise.

We further study the asset-pricing implications of the disaster risk, disaster, and other states of the economy on risk-free debt markets. A higher likelihood of a disaster will decrease the marginal rate of substitution between today's and tomorrow's consumption, thus pushing down the short rate, and causing flight to safety. Instances of disasters change the wealth distribution, and can lead investors to update their beliefs. We show that a rising share of pessimists' wealth in the economy and a smaller belief dispersion - both of which frequently occur in the wake of a disaster - also result in a lower short rate. We extend our discussions beyond the effects on the instantaneous risk-free rate to address the impacts on the term structure of the risk-free rate. Disaster risk and other economic states exert substantial influence on the shape of the risk-free yield curve. The influence over short maturity (say, less than two years) can be different from the influence over long maturity (say, more than twenty years).

Similarly, we also study the impact of disaster risk on credit spreads and their term structures. Due to the possibility of a jump to default, the instantaneous credit spreads are positive, and increase with the disaster risk. The effects of a disaster on credit spreads also depend on investors' beliefs and wealth distribution. A rising share of pessimists' wealth in the economy results in larger credit spreads, but smaller belief dispersion reduces credit spreads. More importantly, our analysis on the risky bond market links the reduced-form model of the term structure of credit spreads in Duffie and Singleton (1999) to the underlying economic mechanisms at work, characterizing rich dynamics between states of the economy and the yield curve of risky bonds.

In summary, our paper thoroughly analyzes the effects of disasters on credit markets, demonstrating that the impacts are contingent on market leverage and default on debt.

To our knowledge, it is the first paper that studies the impacts of disasters on the price and quantity of risky and risk-free debt jointly. The richness of the model produces many implications in line with stylized empirical facts. In addition to revealing a positive relationship between the amount of risky and risk-free debt, for instance, the model also produces a correlation coefficient between credit spreads and economic leverage consistent with its empirical counterpart - a feature that the standard trade-off model fails to account for.

Our paper contributes to a recent literature on rare disaster risks. Many papers have studied the impacts of disaster risk on financial markets and the real economy.[1] Our paper most closely related to Dieckmann (2011) and Chen et al. (2012). Chen et al. (2012) study the impact of disagreement among agents regarding the likelihood and severity of rare disasters on the risk premium and risk sharing. Dieckmann (2011) compares the asset-pricing implications of the disaster risk in complete markets and incomplete markets. By contrast, we focus on credit markets instead of stock markets. Our paper has two other major distinctions: First, the complete markets and incomplete markets in Dieckmann (2011) are two extreme cases, i.e., investors can perfectly hedge against the disaster risk with disaster insurance in the complete markets, and they cannot hedge at all in the incomplete markets. By contrast, we discuss an incomplete market arguably closer to the reality in which investors can imperfectly hedge against disaster risk with risky debt. The ability to perfectly hedge against disaster risk is challenging in practice, but the default possibility embedded in the risky debt offers partial insurance against the disaster risk to investors. Second, and more importantly, our study introduces risky debt, and thus highlights the heterogeneous effects of disaster risk on different segments of credit markets. Previous studies on disaster risk have exclusively considered risk-free

[1] Related work on rare disaster risk includes: Rietz (1988), Liu et al. (2005), Gourio (2008), Farhi et al. (2009), Santa-Clara and Yan (2010), Gabaix (2011), Backus et al. (2011), Barro and Jin (2011), Gourio (2012), Julliard and Ghosh (2012), Nakamura et al. (2013), Seo and Wachter (2013), Kelly and Jiang (2014), and Farhi and Gabaix (2016), among others.

debt. In these studies, however, debt is assumed to have characteristics similar to those of risky debt, i.e., risk-free debt has an exogenous default probability when a disaster occurs. Separating risky debt from risk-free debt enables us to connect debt default to the underlying economy. Moreover, it reveals the interconnection between the two segments of credit markets, and offers new insights on the deleveraging issues that are likely to emerge in the wake of disasters, such as COVID-19.

The remainder of this paper proceeds as follows: Section 1 outlines the model. Section 2 presents a single-agent version of the model as a benchmark, and accentuates the role of risky debt in risk sharing. Section 3 characterizes the equilibrium of the two-agent model. Section 4 discusses the impacts of a rare disaster on debt default and wealth redistribution, and addresses how these issues are contingent on the state of the economy. Section 5 analyzes the asset-pricing implications of disaster risk and other states of the economy on the credit markets. Section 6 discusses the survival of investors in the long run, and other forms of disagreement. Section 7 concludes. All proofs are available in the appendix.

1 Model

1.1 Economy Fundamentals

This section lays out the basic set-up for the model. We consider a pure-exchange economy. The economy is endowed with a flow of a single perishable consumption good, which also serves as the numeraire. The aggregate endowment $\mathscr{E}_t$ follows a geometric Brownian motion with Poisson jump,

$$\frac{d\mathscr{E}_t}{\mathscr{E}_{t-}} = \left(\mu_t - \lambda_t \mathbb{E}\left[e^Y - 1\right]\right) dt + \sigma dz_t^{\mathscr{E}} + (e^{Y_t} - 1)\, dN_t \tag{1}$$

where μ_t is the time-varying expected growth rate of the aggregate endowment. σ is the constant volatility; $z_t^{\mathscr{E}}$ is a standard Brownian motion; and N_t is a Poisson process with intensity λ_t. $k_t := e^{Y_t} - 1$ is the stochastic jump amplitude of the rare-event risk. Details on μ_t, λ_t and $k_t := e^{Y_t} - 1$ are elaborated below.

Jump intensity λ_t follows a Cox–Ingersoll–Ross (CIR) stochastic process

$$d\lambda_t = \alpha_\lambda(\bar{\lambda} - \lambda_t)\,dt + \sigma_\lambda\sqrt{\lambda_t}dz_t^\lambda \tag{2}$$

with unconditional mean $\bar{\lambda}$ and stationary variance $\frac{\bar{\lambda}\sigma_\lambda^2}{2\alpha_\lambda}$. α_λ is the mean-reversion parameter; σ_λ is the volatility parameter; and z_t^λ is a standard Brownian motion independent of $z_t^{\mathscr{E}}$. To preclude λ_t ever being zero, the parameters have to satisfy the condition: $2\alpha_\lambda\bar{\lambda} \geq \sigma_\lambda^2$ (Cox et al., 1985).

Jump amplitude $k_t := e^{Y_t} - 1$ represents the instantaneous drop or boom of aggregate endowment upon arrival of the rare event (Tsai and Wachter (2015)). $\{Y_i\}$ are independent and identically distributed random variables, and they follow a generalized logistic distribution on the real line with probability density function (pdf) given by

$$p_Y(y) = \frac{1}{\mathscr{B}(2,2)}\frac{e^{-2y}}{(1+e^{-y})^4}, \, y \in (-\infty, \infty) \tag{3}$$

where $\mathscr{B}$ is the Beta function.The average disaster size implied by the distribution is

$$\int_{-\infty}^{0}(e^Y - 1)p_Y(y) = -0.25,$$

similar to the 23% figure as calculated by Barro and Ursúa (2008) based on the international data on large consumption declines. The generalized logistic distribution has a thinner tail compared to the standard one, and, as shown in Section 3, offers a closed-form characterization of equilibrium credit spreads.

The time-varying expected growth rate of the endowment μ_t follows a mean-reverting process whose dynamics are given by

$$d\mu_t = \alpha_\mu(\bar{\mu} - \mu_t)\,dt + \sigma_\mu\,dz_t^\mu \tag{4}$$

α_μ, σ_μ are mean-reversion and volatility parameters, respectively; $\bar{\mu}$ is the unconditional mean of μ_t; and z_t^μ is a standard Brownian motion independent of $\{z_t^{\mathscr{E}}, z_t^\lambda\}$. μ_t is unknown to the agents, but all other parameters are public information. However, the agents can learn μ_t from endowment dynamics (1) and (4). Nevertheless, as shown below, the agents display behavioral bias during learning, and, thus, the model generates time-varying beliefs dispersion endogenously.

1.2 Learning and Inference

Since μ_t is unknown, agents have to learn and make an inference about the true underlying parameter μ_t. Their learning, however, could be influenced by behavioral bias such as overconfidence, leading to heterogeneous beliefs and trading (Harrison and Kreps (1978)). Evidence suggests time-varying belief dispersion across agents. For instance, the Survey of Professional Forecasters issued by the Federal Reserve Bank of Philadelphia shows that the belief dispersion for forecasts of annualized real consumption growth of the next quarter ranged from 0.35% to 5.21% with a standard deviation of 0.7% between 1981 to 2019.

Our paper follows Scheinkman and Xiong (2003) to model time-varying heterogeneous beliefs. We assume that there are two types of agents in the market, A and B. Agents form their beliefs from learning information. In addition to receiving public information disclosures (1) and (4), they each respectively also receive a signal regarding μ_t , s_t^A and s_t^B, which follows

$$ds_t^A = \mu_t\, dt + \sigma_s\, dz_t^A \tag{5}$$

$$ds_t^B = \mu_t\, dt + \sigma_s\, dz_t^B \tag{6}$$

Without loss of generality, we assume that $\{z_t^{\mathscr{E}}, z_t^A, z_t^B, z_t^\lambda, z_t^\mu\}$ are Brownian motions independent of each other. Both types of agents know each other's signals. Nevertheless, both types display overconfidence towards their own signals, and exaggerate them when learning. Specifically, agent A perceives s_t^A as

$$ds_t^A = \mu_t\, dt + \phi\sigma_s\, dz_t^\mu + (1-\phi)\,\sigma_s\, dz_t^A \tag{7}$$

meaning that agent A (falsely) believes that the innovation of s_t^A is correlated with the innovation of μ_t. A similar bias occurs to B, too. Agent B perceives s_t^B as

$$ds_t^B = \mu_t\, dt + \phi\sigma_s\, dz_t^\mu + (1-\phi)\,\sigma_s\, dz_t^B \tag{8}$$

The learning problem falls into the wider category of optimal filtering problems, which have been studied extensively in the literature (Liptser and Shiryaev (2001)). The agent's posterior distribution about μ_t conditional on all information up to time t follows a normal distribution

$$\mu_t \sim N\left(\tilde{\mu}_t^i, v_t^i\right), i \in \{A, B\} \tag{9}$$

Since μ_t is a time-varying process, in general, each type of agent will never learn the true value perfectly, and there exists a steady state v^* for $v_t^i, i \in \{A, B\}$,

$$v^* = \frac{\sqrt{\left[\alpha_\mu + \left(\frac{\phi\sigma_\mu}{\sigma_s}\right)\right]^2 + (1-\phi^2)\left[2\frac{\sigma_\mu^2}{\sigma_s^2} + \frac{\sigma_\mu^2}{\sigma^2}\right]} - \left[\alpha_\mu + \left(\frac{\phi\sigma_\mu}{\sigma_s}\right)\right]}{(\frac{1}{\sigma})^2 + \frac{2}{\sigma_s^2}} \tag{10}$$

The mean $\tilde{\mu}_t^A$ of agent A follows

$$\begin{aligned} d\tilde{\mu}_t^A = & -\alpha_\mu\left(\tilde{\mu}_t^A - \bar{\mu}\right)dt + \frac{\phi\sigma_s\sigma_\mu + v^*}{\sigma_s^2}\left(ds_t^A - \tilde{\mu}_t^A\,dt\right) \\ & + \frac{v^*}{\sigma_s^2}\left(ds_t^B - \tilde{\mu}_t^A\,dt\right) + \frac{v^*}{\sigma^2}\left(d\ln\left(\tilde{\mathscr{E}}_t\right) - \tilde{\mu}_t^A\,dt\right) \end{aligned} \tag{11}$$

The mean $\tilde{\mu_t^B}$ of agent B follows an isomorphic process.

Our paper focuses on heterogeneous beliefs of $\tilde{\mu}_t^A$ and $\tilde{\mu}_t^B$, and thus assumes that the agents start with stationary variance $v_0 = v^*$. Also, to facilitate the discussion, let

$$\mu^o = \max\{\tilde{\mu}^A, \tilde{\mu}^B\} \tag{12}$$

$$\mu^p = \min\{\tilde{\mu}^A, \tilde{\mu}^B\} \tag{13}$$

Therefore, μ^o and μ^p are the beliefs of "optimists" and "pessimists," respectively.

The behavioral bias generates time-varying beliefs dispersion and, thus, active trading among the agents. Note that the two types of agents display symmetric behavioral biases, i.e., the bias parameter ϕ is the same across two types of agents, and, on average, neither agent has an advantage over the other. Xiong and Yan (2010) show that both types of agents are able to survive in the long run if the markets are complete. Nonetheless, Section 6 shows that incomplete hedge against the disaster risk will affect agents' survival - and, in fact, will favor the creditors.

1.3 Financial Markets

As the jump amplitude of the endowment follows a continuous distribution, a perfect hedge against the jump risk and, hence, a complete market require a continuum of spot insurance contracts, each corresponding to a possible jump amplitude (Chen et al., 2012).[2] Instead, our financial markets are comprised of three types of assets: stocks, risk-free debt, and risky debt. Hence, the market is incomplete. The incomplete market, however, is different from the one in Dieckmann (2011), which consists of only stocks and risk-free debt. As it becomes clear as we progress, the risky debt facilitates risk-sharing between investors, and complements the risk-free debt.

Stock S is the claim to aggregate endowment. The total number of shares of the stock in the economy is normalized to one.

Safe Debt B^f is an instantaneous risk-free zero-coupon bond that follows

$$\frac{dB_t^f}{B_{t-}^f} = r_{t-}^f \, dt \tag{14}$$

where r_{t-}^f is the risk-free interest rate. Note that since the bond B^f is absolutely risk-free, it does not allow any form of default. Thus, the bond can be regarded as the "safe asset" in Barro and Mollerus (2014).

Risky Debt B^d is an instantaneous zero-coupon but defaultable bond. Bond issuers can default on the debt if they are not able to repay the principal or interest. Bond issuers borrow B_{t-}^d at $t-$, and promise to repay principal B_{t-}^d and interest $B_{t-}^d r_{t-}^d$ at $t+dt$, *conditional on them not defaulting at* $t+dt$. Precisely,

$$\frac{dB_t^d}{B_{t-}^d} = r_{t-}^d \, dt + k_t^d \, dN_t \tag{15}$$

where r_{t-}^d is the expected return on the risky debt, and k_t^d is the write-down of its principle when default is triggered.

[2] Jones (1984) shows that if an underlying asset's jump size has a finite state distribution, a sufficient number of different contingent claims written on this asset can help to fully hedge jump risk and complete the market.

To complete the risky debt characterization, we make the following assumptions:

Assumption 1.1 *The default occurs when the issuer's wealth suddenly drops by more than* $|\gamma|, -1 < \gamma < 0$.

Assumption 1.1 echoes Black and Cox (1976) and Longstaff and Schwartz (1995), resembling the situation that occurred in the financial crisis when borrowers suffered deep losses, and were unable to repay debts. γ can be regarded as the net worth shock that triggers the debtor's decision to walk away from the debt.[3]

Assumption 1.2 *The write-down* $k_t^d = \eta k_t, \eta \in (0,1]$ *when default is triggered.*

Assumption 1.2 says that the write-down proportionally moves with the market when there is a default on debt. Similarly, Barro (2006) assumes the loss given default equal to the size of economic contraction. Since the default can only occur when the rare event N_t materializes, a high write-down k_t^d reflects the difficulty of recovering the bond value when the market experiences a deep plunge. This approach essentially models a stochastic recovery of face value of the bonds as assumed in Duffie and Singleton (1999).

Given the assumptions, it is easy to see that k_t^d of the risky debt depends on an endogenously determined default threshold $\bar{k}$:

$$k^d = \begin{cases} 0, & \text{if } k > \bar{k} \\ \eta k, & \text{if } k \leq \bar{k} \end{cases} \tag{16}$$

The realization of a rare disaster does not necessarily trigger default. Default only occurs when $k = e^Y - 1$ is lower than the threshold $\bar{k}$, i.e., when the downward jump is sufficiently

[3] Alternatively, $1+\gamma$ is the proportion of the net worth that cannot be seized by creditors. It can be interpreted as the minimum proportion of the net worth kept as a social safety net.

large.[4] Intuitively, the default threshold $\bar{k}$ should closely link to the investors' leveraged positions.

Finally, the preferences of the two classes of investors are

$$\mathbb{E}^i\left[\int_0^\infty e^{-\rho t}\log(C_t^i)\,dt\right], i \in \{o,p\} \tag{17}$$

$\mathbb{E}^i$ is the expectation with respect to the belief of each type of agent. C_t^i denote the total consumption of each class of investor. In other words, two classes of investors are identical except for holding different beliefs. Investors in this economy can trade competitively in the securities market, and consume the proceeds. The investor's wealth process conforms to the stochastic differential equation

$$\frac{dW^i}{W^i} = \theta^i\left(\frac{dS}{S} + \frac{\mathscr{E}}{S}\right) + \theta^{d,i}\frac{dB^d}{B^d} + (1-\theta^i-\theta^{d,i})\frac{dB^f}{B^f} - c^i, i \in \{o,p\} \tag{18}$$

where θ^i and $\theta^{d,i}$ are optimist's positions on stock and risky debt, and $c^i := \frac{C^i}{W^i}$ is the consumption-wealth ratio.

Market equilibrium in this economy consists of a pair of price processes $\{r_t^d, r_t^f, S_t, \bar{k}_t\}$ and the consumption-trading strategies $\{\theta_t^i, \theta_t^{d,i}, c_t^i\}$, $i \in \{o,p\}$ such that the investors' expected lifetime utilities are maximized subject to their respective wealth dynamics in Equation

[4] Similar to the exposition of Merton (1974), the risky debt B^d can be regarded as a safe debt minus an option (i.e., a down-and-in option). The payoff at $t+$ is $B^d - \mathbb{1}_{\{t+\}}\mathbb{1}_{\{k\leq\eta\bar{k}\}}|k|$, where $\mathbb{1}_{\{t+\}}$ is an indicator variable equal to one when there is a downward jump and zero otherwise and $\mathbb{1}_{\{k\leq\bar{k}\}}$ is another indicator variable equal to one when the jump is less than or equal to $\bar{k}$, and zero otherwise.

(18). The securities markets clear:

$$\theta_t^o W_t^o + \theta_t^p W_t^p = S_t \tag{19}$$

$$\theta_t^{d,o} W^o + \theta_t^{d,p} W_t^p = 0 \tag{20}$$

$$(1 - \theta_t^o - \theta_t^{d,o}) W_t^o + (1 - \theta_t^p - \theta_t^{d,p}) W_t^p = 0 \tag{21}$$

And, in addition, $\bar{k}_t = \frac{\gamma}{\theta_t^o}$ in equilibrium. Hitherto, we will omit the subscript t denoting time t.

2 The Single-agent Equilibrium

Before moving to the results for the two-agent model,we first review the trading and asset-pricing implications of the single-representative-agent model in our setting. The results from the single-agent model then provide a benchmark for comparison to those from the two-agent model.

The single-agent model is nested within the two-agent model by assuming that only one of the two agents (e.g., the optimists) is present in the market. Thus, the optimists are initially endowed with all of the shares outstanding. To highlight the critical role played by the risky debt B^d, we exclude the risky debt from the financial market by restricting $\theta^{d,o} = 0$. The optimists then maximize their expected lifetime utility through consumption and investment choices on stocks and risk-free debts. The market clearing conditions imply that the investors allocate all their wealth into the stocks and none into the safe debt. Namely, $\theta^o = 1$. Because only optimists populate the market, and because they hold stocks exclusively, no active trading occurs in the market. Lemma 2.1 summarizes the single-agent equilibrium:

Lemma 2.1 *In the single-agent economy, the equilibrium stock price is given by*

$$S = \frac{\mathscr{E}}{\rho} \tag{22}$$

and the risk-free rate is

$$r^f = \mu^o + \rho - \sigma^2 - 2\lambda \tag{23}$$

Market clearing conditions imply that investors do not borrow in equilibrium. Still, we can conclude that they are unwilling to borrow and are reluctant to take leveraged positions without considering market clearing conditions. Note that $k = e^Y - 1$ in (1) has support on $(-1, \infty)$. The aggregate endowment has a risk of dropping to a positive yet arbitrarily small amount. Given the stock price in (22), any non-trivial leveraged position (i.e., the situation in which an optimist borrows a bit to purchase stock) would result in negative wealth with positive probability, i.e.,

$$\mathbb{P}(\theta^o k < -1) > 0 \tag{24}$$

which is inadmissible for the log utility. Thus, they will do their best to maintain positive wealth and keep $\theta^o \leq 1$. This insight, when applying to the two-agent economy, implies that neither type of investor will borrow. Intuitively, the "absolutely safe" debt is too much to ask for in our economy subject to the disaster risk, and it does not accommodate risk sharing between investors. In this regard, the risky debt emerges endogenously in the economy, and it facilitates risk sharing. As shown below, the optimists would issue risky bonds B^d to finance their leveraged positions, and the pessimists would issue safe bonds B^f in equilibrium.

3 The Two-agent Equilibrium

In this section, we characterize the two-agent equilibrium. Note that Lemma 2.1 shows that the stock price is a multiple of the aggregate endowment, irrespective of the investors' beliefs. Therefore, we conjecture that the stock price remains the same in the two-agent economy. We eventually verify this in the equilibrium. The optimists' first-order conditions yield

$$\mu^o + \rho - \lambda \mathbb{E}[k] - r^f - \theta^o \sigma^2 + \lambda \mathbb{E}\left[\frac{k}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{25}$$

$$r^d - r^f + \lambda \mathbb{E}\left[\frac{k^d}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{26}$$

Since investors concur on the disaster risk, including its intensity and size distribution, we drop superscript i denoting agent $i's$ expectation.

Given risky bonds considered in equations (15) and (16), we obtain the credit spreads as

$$r^d - r^f = -\underbrace{\lambda}_{\text{Probability of jump}} \underbrace{\mathbb{P}\left(k \leq \bar{k}\right)}_{\text{Probability of severe jump}} \underbrace{\mathbb{E}\left[\frac{\eta k}{1 + \left(\theta^o + \theta^{d,o}\eta\right)k} \middle| k \leq \bar{k}\right]}_{\text{Expected LGD under risk-neutral probability}} \tag{27}$$

Note that $\frac{1}{1+\left(\theta^o+\theta^{d,o}\eta\right)k}$ is the pricing kernel conditional that debt default occurs at t. Equation (27) characterizes the credit spreads on the risky bonds as the product of three components: the probability of a disaster occurring, the probability of the disaster triggering default, and the loss given default (LGD) under risk-neutral probability.

Theorem 3.1 establishes the equilibrium result.

Theorem 3.1 *Let* $\omega := \frac{W^o}{W^o+W^p}$ *be the optimist's relative wealth share in the economy and* $\mathscr{Q}(\theta^o)$ *be a continuous function of* $\theta^o \in [1, \frac{1}{\omega}]$ *whose specific form is presented in the appendix.*

1. *Stock price* $S = \frac{\mathscr{E}}{\rho}$

2. *When* $\mu^o - \mu^p < \max\limits_{\theta^o \in [1, \frac{1}{\omega}]} \mathscr{Q}(\theta^o)$, *the optimist's position on stock* θ_*^o *is the solution to the equation* $\mu^o - \mu^p = \mathscr{Q}(\theta^o)$, *and the pessimist's position on stock is* $\theta_*^p = \frac{1-\omega\theta_*^o}{1-\omega}$. *Otherwise,* $\theta_*^o = \frac{1}{\omega}$ *and* $\theta_*^p = 0$

3. *The investors' positions on risk-free and risky debt follow*

$$\frac{1-\theta^o-\theta^{d,o}}{\theta^{d,o}} = \frac{1-\theta^p-\theta^{d,p}}{\theta^{d,p}} = \eta - 1 \tag{28}$$

4. *The credit spread on* B^d *is*

$$r^d - r^f = -\eta\lambda_t\left(\frac{4}{(\bar{k}+2)^3} - \frac{3}{(\bar{k}+2)^2} - 1\right) \tag{29}$$

where $\bar{k} = \frac{\gamma}{\theta_*^o}$

Theorem 3.1 shows that when the dispersion of beliefs becomes large enough, the credit spread $r^d - r^f$ is solely determined by the disaster risk and optimists' relative wealth share, regardless of the dispersion of beliefs between the two types of investors. This is due to the fact that the pessimists face a natural "no-short-sale" constraint. Note that the economy is also subject to positive jumps. Once the pessimists start to short shares, they risk negative wealth upon arrival of an upward jump. Hence, after belief dispersion increases over some threshold, although the optimists would like to obtain more shares, they have exhausted all the shares they could possibly obtain in the market. Under such circumstances, therefore, a larger belief dispersion leads to neither higher leverage nor higher credit spreads, but a heightened risk-free rate instead.[5]

[5] Miller (1977) argues that when investors have heterogeneous beliefs about stock fundamentals, a "no short sale" constraint can cause stock to be overpriced. Harrison and Kreps (1978) and Scheinkman and Xiong (2003) show that an optimist would like to pay more than his own expectation of the asset's fundamental because of the resale option in the future. Theorem 3.1 shows that such an effect does not arise here. Part of the reason is that the rise of the risk-free rate and credit spreads makes leverage costly in general equilibrium. Part of the reason is that logarithmic utility is myopic.

Theorem 3.1 also highlights an interesting relationship between the risky debt and risk-free debt. In equilibrium, the optimists take leveraged position by issuing risky debt and purchasing risk-free debt at the same time. Correspondingly, the pessimists finance the optimists' positions by purchasing risky debt and issuing risk-free debt at the same time. The ratio between the amount of the two types of debt is determined by loss-sharing parameter η. By issuing risky debt and holding risk-free debt, the optimists de facto assemble a disaster insurance. Equation (28) suggests that for one unit of issued risky debt, the optimists will purchase $1-\eta$ units of risk-free debt. The return on the debt portion of their portfolios

$$(1-\eta)\frac{dB^f}{B^f} - \frac{dB^d}{B^d} = \left((1-\eta)r^f - r^d\right) dt - k^d\, dN_t \tag{30}$$

The optimists pay $r^d - (1-\eta)r^f = \eta r^f + (r^d - r^f)$ for the write-down k^d when a disaster materializes and they cannot afford to pay off the debt. They are willing to pay more if the loss sharing embedded in the risky debt is more generous, i.e., if η is greater. The relationship between risky debt and risk-free debt highlighted by Equation 28 helps explain a conjecture by Friedman (1981) on federal debt and private sector debt. We elaborate on this further in Section 5.3.

4 Disaster, Default and Credit Markets

Instances of disasters can give rise to default on debt, and disrupt credit markets. Nonetheless, Theorem 3.1 shows that default does not necessarily occur. It shows, instead, that default depends on several factors, including belief dispersion, disaster likelihood, and wealth distribution among investors. How do these factors affect the possibility of default? What are the impacts of disasters and debt default on credit markets? In this section, we formally explore these questions.

4.1 Model Parameters

To facilitate our analysis, we use a set of calibrated parameters given in Table 1. We divide the model parameters into three groups: economy fundamentals, beliefs formation and preference, and rare-event risk. The details of the calibration procedure are explained below.

Aggregate endowment dynamics Four parameters, $\bar{\mu}$, $\sigma, \alpha_\mu, \sigma_\mu$, determine the aggregate endowment dynamics during normal times. $\bar{\mu}$ is set equal to 1.55% to match the mean growth rate of US aggregate consumption from 1990 to 2019 . The remaining three parameters are chosen as follows: the volatility of endowment growth $\sigma = 3.44\%$; the volatility of endowment expected growth $\sigma_\mu = 1.1\%$; and the mean reversion $\alpha_\mu = 0.05$. Following Brennan and Xia (2001), these parameters are calibrated to match the values of three empirical moments: $Var(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-1})), Cov(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-1}), \log(\mathscr{E}_{t-1}) - \log(\mathscr{E}_{t-2})), Cov(\log(\mathscr{E}_t) - \log(\mathscr{E}_{t-2}), \log(\mathscr{E}_{t-1}) - \log(\mathscr{E}_{t-3}))$, i.e., the variance of per capita consumption, and the first- and second-order autocorrelations of consumption growth over 1990-2019.

Beliefs formation and time preference The signal volatility σ_s and agents' overconfidence bias ϕ affect the formation of beliefs. These parameters are chosen to match the mean and volatility of beliefs dispersion in the Survey of Professional Forecasters (SPF) by the Philadelphia Federal Reserve Bank. The average belief dispersion for annualized real consumption growth is 1.11%, and the volatility is 0.80%, calculated using data from 1990 to 2019. The time preference parameter ρ is 0.03. This parameter value has been used in the savings literature, such as in Hubbard et al. (1995).

Rare-event risk Due to the rare nature of disasters, it is challenging to precisely estimate the probability that one will occur. The unconditional mean $\bar{\lambda}$ is set to be 1.169% per annum. Current literature (e.g., Barro and Ursúa (2008), Wachter (2013) and Seo and

Wachter (2016)) has calibrated the long-run mean of rare-event risk $\bar{\lambda}$ to values between 2% and 3.55% per annum. So, our calibration of the unconditional mean of rare-event risk is conservative. The volatility σ_λ is set at 0.081, consistent with Wachter (2013). The mean reversion parameter α_λ is set at 0.11. This number implies that it takes $\frac{\log(2)}{\alpha_\lambda} = 2.74$ years for the difference between the disaster risk intensity and its long-run mean to converge by half. Note that although the paper's primary focus is credit markets, these parameter values have been used in various work in the literature to account for several puzzles in the stock market- such as the equity premium puzzle, the excess volatility puzzle, the value premium anomaly, and aggregate stock market predictability.

The model is eventually left with two free parameters, η and γ (i.e., the write-down parameter of the risky debt and the net-worth loss that forces optimists to default). Equation (28) implies that the ratio between federal debt (risk-free) and other types of risky debt can calibrate η. Using the longest data series from Federal Reserve Board, we calculate the ratio to be 0.306, and, hence, we set $\eta = 0.694$. Finally, the paper sets $\gamma = -0.95$ (i.e., the leveraged investors will default on debt once their net equity remains 5% or less). Admittedly, without detailed granular data, it is difficult to determine the magnitude of the default-triggering loss.

Table 2 presents the moments of risk-free rates and credit spreads from the simulation results. Following the convention in the rare-event literature, we simulate monthly data for 10,000 50-year sample paths. For each statistic, we report population values, percentile values from the small-sample simulations, and percentile values from the small-sample simulations in which rare disasters do not occur. The empirical moments of risk-free bonds are constructed from the three-month Treasury Bill data, while the empirical moments of risky bonds are constructed from the Bank of America AA US Corporate Index Option-Adjusted Spread. Compared to previous literature, the model accounts not only for the variation of risk-free rates, but also successfully replicates the first and second moments of the risky bonds. The average credit spreads are 100 basis points(bps), and the volatility is 67.87 bps, both of which fall in the 90% confidence interval predicted by the model.

Although realizations of disasters do not substantially change the unconditional moments, their impacts on the credit markets are profound, and depend heavily on the states of the economy, as Section 4.2 shows.

4.2 Disaster and Loss Given Default

As an example, Table 3 shows the loss given default as a fraction of the total wealth for different states of the economy when the endowment suddenly drops by 50%. The first line features the baseline parameter values. The default and write-down occur if the disaster size is greater than $\bar{k} = 49\%$. When the disaster materializes, holders of the risky bonds write off a portion of their debt, amounting to 23.43% of total wealth.

Table 3 indicates that different states of the economy would result in different degrees of loss for the same size of a disaster. The amount of loss increases with belief dispersion, but decreases with the share of wealth held by optimists, and with the intensity of the disaster. Note that when belief dispersion becomes 0.65%, or when the optimist's wealth share rises to 70%, the loss is zero. This is because the default thresholds $\bar{k}$ are less than -50% under those circumstances. Each type of investor solely absorbs the loss on the stock investments, and the leveraged borrowers do not have to default on the debt. In sum, although the magnitude of loss may depend on the specific risky bonds we consider in (15) and (16), the results in Table 3 highlight that the disruption to credit markets by a disaster depends on whether the disaster induces default on debt.

Moreover, Table 3 also shows that the disaster's impact on the wealth distribution depends on whether default occurs. The realization of a disaster always decreases the optimists' wealth share in the economy. The reduction, nonetheless, is more substantial when the optimists cannot default on their debt. For example, if they own 70% of the total wealth, two-thirds of their wealth will be wiped out when the aggregate endowment suddenly drops by 50% and they have to absorb all the loss on their own. Default on the risky bonds, nevertheless, allows them to share part of the loss with creditors, alleviating the wealth loss caused by the disaster. As Theorem 3.1 shows that wealth distribution is a key determinant of the price and quantity of the credit, the post-disaster credit markets also hinge on whether incidents of disaster cause debt default.

The key to different impacts of the same disaster lies in the optimists' leverage. Disasters do not necessarily lead to default loss. As illustrated in Section 2, even though the endowment suddenly and deeply plummets, debt default does not occur if the investors avoid leverage ex-ante. The leverage is a result of the states of the economy, including belief dispersion, disaster intensity, and wealth distribution. Figure 2 illustrates the effects of state variables on the optimist's leveraged position θ^o and quantities of different debt relative to the total wealth in the economy.

Panel A plots the optimist's leveraged position and debt quantities as functions of disaster intensity. As the likelihood of a disaster increases, optimists rapidly reduce their leveraged positions. Consequently, the debt share in the economy also drops. If the disaster materializes when the jump intensity is high, the direct disruption to the credit market will be less severe as investors are already taking it into account. Panel A also shows that the rate of deleveraging is not constant, but changes with the likelihood of disaster. Initially, investors decrease the leverage slowly, but this accelerates with disaster risk.

Panel B plots the optimists' leveraged position and debt quantities against the optimists' wealth share. Although the optimists' portfolio weight on risky asset decreases with their wealth share, the total debt share in the economy is non-monotonic. As the optimists possess more of the wealth of the economy, the weight of risky assets in their portfolios falls, and thus their demand for leverage also drops. Yet, since their wealth share in the economy increases, the total amount of risky bonds they issue might rise. The two competing effects result in a non-monotonic relationship between the debt quantities and the optimists' wealth share. The non-monotonic relationship also creates a divergence between private and social impacts of default caused by a disaster. For example, when the pessimists possess most of the wealth in the economy, the optimists will take on excessive leverage, exposing themselves to greater disaster (and, hence, default) risk and potential loss. Yet, the loss given default might be moderate from a social perspective.

Similarly, Panel C plots the optimists' leveraged position and debt quantities as functions of belief dispersion. As belief dispersion widens, the optimists start to take on more leverage, and to purchase more risky assets. As a result, the total debt share in the economy climbs. Hence, when belief dispersion grows wider, the credit market is more susceptible to a disaster and the total exposure of the debt to default risk is large.

5 Disaster and Credit Pricing

Disaster risk affects not only investor trading (as shown in Section 4.2), but the price and quantity of credit. In this section, we study the asset-pricing implications of disaster risk on credit markets, including both risk-free and risky bond market segments. For each segment, we first focus the instantaneous expected return, and then move to term structure of the interest rate and credit spreads.

5.1 Risk-free Bond Market

5.1.1 Instantaneous Risk-free Rate

Figure 3 illustrates the behavior of instantaneous interest rate under baseline parameter values. Panel A plots the short rate as a function of disaster intensities. For both types of investors, higher likelihood of a disaster will decrease the marginal rate of substitution between consumption today and consumption tomorrow, increasing precautionary saving and pushing down the short rate. The depressed short rate, plus the reduced leverage positions shown in Figure 2 (A), captures the "flight to safety" during market turmoil.

Panel A illustrates the direct impact of the disaster intensity on the short rate. The disaster can also influence the short rate through other channels. First, the realization of a disaster can suddenly change the wealth distribution between the investors, as shown in Table 3, which can also affect the short rate. Second, although the dynamics of belief dispersion in our model are independent of disaster risk, it is probable that disaster risk and its actual occurrence shape investors' beliefs. For example, higher likelihood of a disaster might induce investors to be less optimistic; a sudden loss of wealth due to a disaster might render them more pessimistic. Hence, to obtain an extensive understanding of the disaster risk on the short rate, it is also imperative to study the impacts of wealth distribution and belief dispersion on the short rate.

Panel B of Figure 3 plots the short interest rate as a function of the optimists' wealth share. Note that, r^f reaches the limiting interest rate when only the pessimists are present in the market; similarly, at $\omega = 1$, r^f approaches the limiting interest rate when only the optimists are present. Overall, the short rate increases with the optimists' share of wealth in the economy. However, it is worth pointing out that such a relationship is not monotonic.

Panel C plots the short rate as a function of belief dispersion. An increase in the belief dispersion would increase the risk-free rate. One way to interpret the widened belief dispersion is to consider that the optimists become more optimistic. Therefore, they would like to take on more leveraged positions by issuing more risky bonds. At the margin, risk-free debt is a substitute to the risky debt among the optimists. Therefore, both the return r^d on risky debt and the return r^f on risk-free debt increase.

5.1.2 Term structure of Interest Rate

Having studied the impacts of the state variables on the instantaneous risk-free rate, we turn to the discussion of their influence on the term structure of interest rate. We consider a zero-coupon bond B_τ that pays one dollar in τ years. Its price therefore is

$$B_\tau = \mathbb{E}\left[\exp\left(-\int_0^\tau r_s^f\, ds\right)\right] \tag{31}$$

where $\mathbb{E}$ indicates the expectation under the objective belief. The yield to maturity of a τ-year bond therefore is

$$y_\tau = -\frac{\log(B_\tau)}{\tau} \tag{32}$$

Although a closed form of the yield y_τ is out of reach, the analytical tractability of our model makes the computation quite manageable.

Figure 4 illustrates the direct influence of the disaster risk on the term structure of the interest rate. Panel A plots bond yield curves for maturities from zero to fifty years. The solid line corresponds to the yield curve when the long-run disaster risk $\bar{\lambda} = 1.32\%$. Similarly, the dotted and dashed lines correspond to $\bar{\lambda} = 1.17\%$ and 1.08%, respectively. Higher $\bar{\lambda}$ decreases the bond yields. The magnitude of the reduction appears to be more substantial for long-term bonds, hence leading to a steeper yield curve. Moreover, although overall the yield curves are downward sloping, Panel B zooms in the yield curve for maturities from zero to two years, and highlights the hump shape for short maturities.

Figure 5 features the wealth distribution's impact on the yield curve. The solid line corresponds to the yield curve when the current optimists' wealth share $\omega = 0.1$. The dotted line and dashed line correspond to $\omega = 0.5$ and 0.85, respectively. One immediate conclusion is that increased wealth share of the optimists raises bond yields across maturities, and the effect is greater on long-term bond yields. Panel B also shows that wealth distribution is a critical determinant of the yield curve shape. It appears that the hump shape of the yield curve disappears when the wealth share is below 0.5. Overall, Figure 5 demonstrates that the impacts of disaster risk on the term structure of the interest rate differ depending on the wealth distribution.

Finally, Figure 6 illustrates the effects of average belief dispersion on the yield curve. An increase in ϕ will raise the average belief dispersion between investors. The dotted line corresponds to the baseline case $\phi = 0.61$. The solid and dashed lines delineate cases of $\phi = 0.4$ and $\phi = 0.8$, respectively. Widened belief dispersion raises the bond yields across different maturities. Moreover, panels A and B together show that the term-structure shape between zero and ten years closely depends on the belief dispersion.

5.2 Risky Bond Market

Risky bonds play a more critical role than risk-free bonds in risk sharing for investors with exposures to disaster risk. Section 2 highlights that in the absence of risky bonds, even though risk-free bonds are available to investors, they shun taking leveraged positions for fear of a disaster. The introduction of risky bonds greatly facilitates the risk sharing between investors, and, consequently, the optimists start to take leveraged positions. In this section, we discuss how the state variables, including disaster risk, affect the price and quantity of risky bonds.

5.2.1 Instantaneous Credit Spreads

Figure 7 illustrates the behavior of instantaneous credit spreads under different state variables. Panel A plots the credit spreads as a function of disaster risk intensity. The credit spread is 60 bps when the disaster risk is 0.6%, and quickly increases to 200 bps when the disaster risk becomes 1.8%. Thus, risky credit becomes more expensive as disaster risk rises, consistent with the flight-to-safety phenomenon during the crisis. Note that although increased disaster risk means a higher likelihood for a disaster to materialize, the greater disaster risk itself mitigates the disruption to the credit market caused by an instance of a disaster, since expensive credit reduces the economy-wide leverage.

Panel B plots the credit spreads as a function of the optimists' wealth share. It shows that credit spreads fall with the optimists' wealth share. As optimists' wealth share increases, the risk-free rate rises, and risky credit also becomes more expensive. Consequently, the optimists take on smaller leveraged positions, and credit spreads fall. Notably, when $\omega = 0$, credit spreads are about 140 bps. In other words, the credit spreads do not shrink to zero when $\omega = 0$, i.e., the circumstances in which only the pessimists are present in the economy, and the optimists' wealth share becomes negligible.

The fact that investors still exert significant influence on credit spreads even if their wealth share is negligible echoes the findings of Kogan et al. (2006). In a market that features both irrational and rational investors, they show that the price impact of irrational traders does not rely on their long-run survival, and that irrational traders can have a significant impact on stock prices even when their wealth becomes negligible. Our model highlights that the similar conclusions can be extended to credit markets. As discussed in Section 2, investors are reluctant to take leveraged positions for fear of disaster risk. The presence of risky bonds accommodates risk sharing and enables trading. The pessimists and optimists are the sole purchasers and sellers of risky bonds in the market; thus, the impacts of pessimists and optimists on the pricing of risky bonds are still substantial, even if their wealth share becomes negligible.

Panel C shows the credit spreads as a function of belief dispersion. Generally speaking, as beliefs dispersion widens, the optimists regard credit as cheap from their perspective, and, therefore, they would like to take on more leveraged positions. The higher leverage translates into a higher k (i.e., a smaller downward jump to trigger default), and pushes up the credit spreads. Note that credit spreads stop increasing with belief dispersion when belief dispersion itself becomes sufficiently wide, due to the "no-short-sale" constraint faced by the pessimists.

5.2.2 Credit Spreads and Leverage

Johnson (2019) finds that while the standard trade-off model can do a reasonable job matching many real and financial moments, it has difficulty explaining the correlation between credit spreads and leverage at the macro level. In the data, the correlation between leverage and credit spreads is positive and robust to different measures of credit spreads and leverage. However, when fitted to data, the model yields a negative correlation. To reconcile the discrepancy between the model and the data, Johnson (2019) introduces default insurance, amplifying the moral-hazard friction embedded in the trade-off model.

In contrast, our paper presents an agency-problem-free model that can also generate a positive association between credit spreads and leverage. In our model, the dynamics of the state variables drive both credit spreads and leverage, generating a positive correlation between the two. For example, a decrease in disaster risk raises both leverage and credit spreads. To quantify the explanatory power of the model, we aggregate our monthly simulations to a quarterly frequency, and compute the correlation using the subset of simulations in which no disaster occurs. The 90% confidence interval of the correlation predicted by the model is $(0.3860, 0.7915)$, and the median is 0.6802, similar to the correlation coefficient (0.5229) calculated by Johnson (2019) using data.

5.2.3 Term structure of Credit Spreads

The explicit introduction of risky bonds enables the discussion of the impacts of the state variables on the term structure of credit spreads. To discuss the impacts, we first define a zero-coupon risky bond $\mathbb{B}^d_\tau$,

$$\mathbb{B}^d_\tau = \int_0^\tau \mathbb{E}\left[\exp\left(-\int_0^t r^f_s\, ds\right)\mathbb{P}(\mathscr{T} = t)\left(1 + k^d_t\right)\right] dt + \mathbb{E}\left[\exp\left(-\int_0^\tau r^f_s\, ds\right)\mathbb{P}(\mathscr{T} > \tau)\right] \tag{33}$$

where $\mathscr{T}$ is the first default time. If $\mathscr{T} > \tau$, $\mathbb{B}^d_\tau$ pays one dollar in τ years. However, if $\mathscr{T} = t \in [0, \tau]$, the bond pays out $1 + k^d_t$ at t immediately, and the bond terminates at $\mathscr{T}$. The credit spreads therefore are

$$y^d_\tau - y_\tau = -\frac{\log\left(\mathbb{B}^d_\tau\right)}{\tau} - y_\tau \tag{34}$$

Duffie and Singleton (1999) build a similar statistical model of term structure of credit spreads in which default is governed by a hazard-rate process, and loss given default is stochastic. Our model rationalizes their reduced-form model in an equilibrium framework, and links it to underlying economic mechanisms at work.

Figure 8 illustrates the disaster risk's impact on the term structure of credit spreads. Note that the credit spread is positive even when the debt maturity approaches zero, due to the jump-to-default risk. Consistent with the intuition, higher $\bar{\lambda}$ shifts the credit spread curve upward. However, compared to the risk-free rate, the effect of higher $\bar{\lambda}$ on the credit spreads is moderate and does not significantly vary across maturities. Although overall the credit spreads rise with debt maturity, Panel B highlights local hump shape for short maturities.

Figure 9 plots the term structure of credit spreads to show the impacts from wealth distribution. Compared to disaster risk, the wealth distribution appears to have more pronounced effects on credit spread curves. Credit spreads rise with the pessimists' wealth share for both short and long maturities, but rise more for longer maturities, leading to a steeper upward slope overall.

Figure 10 illustrates the effects of belief dispersion on the risky bond yield curve. Widened belief dispersion increases credit spreads. For bonds with maturities of less than 20 years, the increase is modest. For the three parameter values considered, credit spreads are below 200 bps, consistent with the data. The belief dispersion also affects the shape of term structure of the credit spreads, especially for short maturities; this is similar to the impact of belief dispersion on the shape of the risk-free bond yield curve. Panel B indicates that, depending on specific parameter values, the credit spread curve can be upward, flat, or hump-shape for short maturities.

5.3 Risky and Risk-free Debt

Although we discuss risky and risk-free debt as two separate sectors of the credit markets, they interact with each other closely. In fact, Friedman (1981) surmised that increases in federal borrowing would curtail private borrowing, either due to the investors' ultra-rationality or credit-market borrowing constraints. This implies a negative association between risk-free debt and risky debt. However, the data display a different picture. As Figure 1 illustrates, the amount of federal debt is positively correlated with the amount of risky debt over time. The correlation coefficient between the amount of two types of debt normalized by GDP (GDP deflator) is 0.83 (0.99) from 1966 to 2008. The result is consistent with Summers (1986), who also found that increases in government debt are actually associated with increases in private debt.

Following Summers (1986), we estimate the relationship between risky debt and risk-free debt with a longer time series. We use three different empirical measures to proxy the risky debt in the model. The first measure is total risky debt, which is the difference between the total amount of debt in the US and the total amount of federal debt outstanding. The second measure is the sum of non-financial business debt and household (including nonprofit organization) credit. The third measure is non-financial corporate business debt.

All measures are normalized by GDP. The data are on a quarterly basis from 1966 to 2019.

Table 4 reports the empirical results. Similar to Summers (1986), we control for one- and two-quarter lags of total federal debt. In addition, we also control for short-term (one-year) and long-term (ten-year) US treasury rates that might affect investors' decisions to take on credit. All results are significant at the 1% level, except for non-financial corporate business debt. The results thus show a strong relationship between federal debt and different measures of risky debt, even after controlling for other macroeconomic variables. In light of the impacts of the 2008 financial crisis and the fiscal and monetary policies that followed, we also estimate the relationship in column (3) using data up to the first quarter of 2008. The coefficient estimate is still significantly positive, and the magnitude remains similar to the magnitude that results when using the full sample.

In fact, our model indicates that federal debt and other types of risky debt are two sides of the same coin. Theorem 3.1 indicates that the ratio of the amount of risk-free debt over total amount of risky debt is determined by the loss sharing between the creditors and debtors, a positive constant $1-\eta$ in the model. Since the optimists always take leveraged positions and have positive default risk, their positions are financed by risky debt. If the loss sharing via the risky debt upon default is not sufficient ($\eta < 1$), their optimal positions also comprise positive holdings of risk-free debt. Hence, the amount of risk-free debt decreases with η. In fact, when $\eta = 1$, their holdings of risk-free debt become zero, and, thus, no risk-free debt is actively traded in the equilibrium. The analysis of trading behaviors between investors also shows that between federal debt and other types of risky debt, the risky debt is the driving force of debt dynamics in the economy.

This result has important policy implications on deleveraging of the economy. The fast-rising national debt, including both public and private debt, has become a major concern in the US. Many studies have argued the consequences of a large and fast-growing debt, such as greater debt-overhang problems, lower national savings and incomes, and greater risk of a fiscal crisis. Yet, when discussing deleveraging, existing studies tend to dichotomize the deleveraging of private sectors and public sectors. By contrast, our results show that public debt and private debt are twin problems; thus, successful deleveraging of either sector is contingent on deleveraging of *both* sectors, calling for a close coordination between the two sectors.

Finally, in addition to accounting for the *quantity* relationship between the risk-free and risky debt, the model also explains the relationship between the risk-free rate and credit spreads. Duffee (1998) provides evidence that the yields on treasury and credit spreads of corporate bonds have a negative relationship. However, the underlying economic mechanisms remain unclear. In our model, both the dynamics of disaster risk and wealth distribution can give rise to such a negative association. We calculate the correlation between the yields of three-month treasury bills and the Bank of America AA US corporate index option-adjusted spread between 1997 and 2019. The result is -0.2625, which falls in the 90% confidence interval of (-0.3821, 0.4378) predicted by the model using the subset of simulations in which no disaster occurs.

6 Model Discussion

6.1 Survival

The survival of investors in the long run is a major focus for models with heterogeneous agents. Borovička (2020) shows that in complete markets, survival chances for agents endowed with separable preferences solely depend on the accuracy of their beliefs. Investors who have more precise beliefs dominate in the long run. If investors have symmetric bias, e.g. equal belief biases with opposite signs, Xiong and Yan (2010) show that both investors are able to survive in the long run if the markets are complete.

However, the conclusion may no longer hold in an incomplete market, such as the one in our model. Table 5 presents summary statistics of the conditional distribution of the optimists' wealth share after 100 years for various initial wealth distributions. Since the realizations of disaster hurt the optimists more, we only focus on the simulation paths in which the disasters do not occur. Although investors in our model have symmetric belief biases, the results show that the optimists' wealth share keeps falling over time, no matter the level of the initial wealth share.

To see the reason, consider that $\eta = 1$ and that only risky bonds are actively traded in the equilibrium. The optimists finance their positions by issuing risky bonds. The cost is

$$r^d = r^f + (r^d - r^f) = r^f + \text{credit spreads} \tag{35}$$

Note that had the economy not been subject to disaster risk, the optimists would only have had to pay the risk-free rate. The credit spreads are premia solely for loss sharing, and for providing insurance against incidents of disaster and possible negative utility. In other words, to take leveraged positions, the optimists now not only have to pay the financing cost r^f but also the extra insurance premia. The credit spreads make the optimists lose wealth relative to the pessimists over time, even though their belief biases are symmetric. This observation also implies that pessimists can still dominate in the economy in the long run even if their beliefs are less precise than those of the optimists.

To confirm, we relax the assumption that two types of investors have the same belief bias ϕ, and we change the optimists' ϕ so that their belief bias is half of that of the pessimists' on average. Figure 11 plots the fitted density of the optimists' wealth share after 100 years, given that the initial share is 0.5. The solid and dash lines correspond to wealth distributions when the two types of investors have symmetric belief biases and when the optimists' belief bias is half of that of the pessimists', respectively. Compared to the symmetric belief biases, the optimists' more accurate beliefs slow down the fall of their wealth share in the market. However, it does not reverse the trend of pessimists dominating the market in the long run. In this regard, incomplete hedge against disaster risk can also affect the credit markets through investors' survival in the long run.

6.2 Disagreement on disaster intensity

Thus far we focused on investors' disagreement on the average growth rate of the economy. Many studies (e.g., Xiong (2013) and Hong and Stein (2007)) have shown that investors' disagreement on the growth rate is substantial, with profound asset-pricing implications. Because disasters materialize infrequently, however, accurately estimating disaster risk is also challenging. This might lead to disagreement on the disaster risk. Chen et al. (2012) study the impacts of the alternative disagreement over the disaster risk when the asset markets are complete. In this section, we explore the impacts when hedging against the disaster risk is incomplete.

To highlight the disagreement on disaster risk, we assume that investors have correct beliefs on the growth rate of the economy. For simplicity, we also assume that $\eta = 1$ and that the disaster intensity is a constant $\bar{\lambda}$. Suppose one type of investor is pessimistic and believes $\lambda^p = \bar{\lambda} = 1.168\%$. The other type of investor is relatively more optimistic and believes it to be $\lambda^o = \frac{\bar{\lambda}}{2} = 0.584\% < \lambda^p$. All other parameter values follow Table 1.

Our results predicated on disagreement over the growth of the economy also hold for disagreement on disaster intensity. Figure 12 shows leverage and asset prices as functions of the optimists' wealth share. Panel A shows that an optimist's position on stocks decrease with the wealth share. Panel B presents the results on asset prices. Similar to Figure 3(B) and 7(B), the risk-free rate rises with the wealth share, while the credit spreads fall at the same time. Figure 13 shows the effects of belief dispersion $\lambda^p - \lambda^o$, as we fix λ^p and adjust

λ^o. Similar to the results in Section 4 and 5, the optimists' positions on stocks rise with the belief dispersion, and so do the risk-free rate and credit spreads.

While the results are largely the same as those based on disagreement over the growth rate, there are several distinctive features. First, in the case of disagreement on the growth rate, wealth distribution does not affect credit spreads directly. It matters only via the optimist's position on stock θ^o. This is no longer true in the case of disagreement on disaster intensity. To see this, note that in Theorem 3.1, r^f is a function of endowment growth μ and disaster risk λ, while credit spread is only a function of λ. In the case of disagreement on the growth rate, given θ^o, both types of investors have no disagreement on credit spreads. The wealth share determines whose belief is present to a greater degree in the risk-free rate. However, in the case of disagreement on disaster risk intensity, the wealth share determines whose belief has more influence on *both* the risk-free rate and credit spreads.

Second, note that according to Theorem 3.1, no risk-free debt is actively traded in the equilibrium if $\eta = 1$. In contrast, Panel A of Figure 12 and Panel A of Figure 13 indicate that risk-free debt is still actively traded when investors disagree on disaster intensity. We have shown that in the case of disagreement over the growth rate, when $\eta = 1$, the optimists are willing to pay $r^d = r^f + \text{(Credit Spreads)}$ for the possible write-down k^d. Nonetheless, in the case of disagreement over disaster risk, the credit spreads depend on pessimists' beliefs, and, hence, credit spreads are expensive in the eyes of optimists ceteris paribus. Holding a positive amount of risk-free debt indicates that they are willing to pay less than r^d for the write-down.

Such willingness to pay for the write-down, however, changes with belief dispersion $\lambda^p - \lambda^o$. Panel A of Figure 13 shows that as optimists become more optimistic and take on more leverage, they would issue more risky debt and hold less risk-free debt, i.e., for one unit of risky debt, they would hold less risk-free debt. This suggests that they are willing to pay more for the write-down in equilibrium as they become more optimistic and take on more leverage. The resultant relationship between the quantities of risk-free and risky debt in Panel A of Figure 13 is contrary to that under disagreement over the growth rate in which the quantities of risk-free and risky debt are proportional and positively associated.

Finally, in the case of disagreement on growth, when the wealth share of either type of investor becomes negligible, the share of different debt in the economy shrinks to zero. This is consistent with the results in Section 2, which indicate that, when only one type of representative investors is present in the market, the equilibrium holdings should be $\theta^o = 1$, and no debt is actively traded. However, this no longer holds when it comes to disagreement on disaster risk. The investors exert pronounced influence on the trading volume of debt. Figure 14 shows that when the optimists' wealth share approaches one (or, equivalently, the pessimists' wealth share approaches zero), the optimists still would like to issue one risky debt and hold one risk-free debt, even though $r^d - r^f$ approaches zero. This is also consistent with Panel A of Figure 13. When belief dispersion $\lambda^p - \lambda^o \to 0$, the optimists issue one risky debt, and the pessimists issue one risk-free debt. This is another example that shows that the limit of a heterogeneous-belief economy does not simply collapse to a representative-agent economy. Heterogeneous beliefs and incomplete hedges against disaster risk have greater implications for asset prices and trading than one could extrapolate from a single-agent model.

7 Conclusion

Disasters such as COVID-19 pandemic have consequential impacts on credit markets. This paper thoroughly studies the impacts of disasters on credit markets. A simple model of heterogeneous beliefs and incomplete hedges against disaster risk yields rich dynamics and implications.

One implication of our model is that incomplete hedging against disaster risk gives rise to a positive association between the amount of federal debt and other types of risky debt. In fact, our model shows that the federal debt and other types of risky debt are two sides of the same coin. Therefore, one sector's successful deleveraging calls for a closer coordination between the two sectors. The implication of this finding is particularly significant in the context of COVID-19 pandemic because it casts light on the effective deleveraging of both public and private sectors- a subject that is likely to be a policy issue in the future.

We show that the degree of credit-market disruption that stems from disasters hinges on whether default occurs in equilibrium. Debt default causes losses to creditors, and affects post-disaster wealth distribution. Whether disasters induce default, however, depends on both the size of the disaster itself and market leverage. Larger belief dispersions, low ex-ante disaster risk, or highly skewed wealth distribution can result in high market leverage, and exacerbate the adverse impacts of disaster instances. We further study the effects of disaster risk on both the quantity and price of credit. We analyze a disaster's impact on the risk-free yield curve and on the term structure of credit spreads. We also reproduce many stylized empirical facts.

Our paper's primary focus is credit markets. Thus, we take a simple approach on the stock market. A general constant relative risk aversion (CRRA) utility or stochastic utility function could highlight the impacts of the disaster on both stock markets and credit markets simultaneously. We leave this topic to future research.

A Solution to the Single-agent Model (Proof to Lemma 2.1)

In equilibrium, $C_t = \mathscr{E}_t$ for the representative agent. Substituting this into the Euler equation gives

$$\frac{S_t}{\mathscr{E}_t} = \int_0^\infty e^{-\rho s} ds = \frac{1}{\rho}. \tag{36}$$

We now derive the instantaneous risk-free rate. Denote the pricing kernel as Λ_t. In equilibrium,

$$\Lambda_t = e^{-\rho t} \frac{1}{C_t} = e^{-\rho t} \mathscr{E}_t^{-1} \tag{37}$$

By Ito's lemma,

$$\frac{d\Lambda_t}{\Lambda_t} = \left(-\mu_t + \lambda_t \mathbb{E}\left[e^Y - 1\right] - \rho + \sigma^2\right) dt + \sigma dz_t^{\mathscr{E}} + \left(e^{-Y} - 1\right) dN_t \tag{38}$$

Hence, the instantaneous risk-free rate r_t^f is

$$r_t^f = -\mathbb{E}_t^o \left[\frac{d\Lambda_t}{\Lambda_t}\right] = \mu_t^o + \rho - \sigma^2 - 2\lambda_t \qquad \square \tag{39}$$

B Solution to the Two-agent Model (Proof to Theorem 3.1)

To derive the equilibrium, we follow several steps:

1. Conjecture stock price $\frac{dS}{S}$ and value function $\mathscr{J}$ and derive first-order conditions.
2. Solve for consumption choices and asset prices from first-order conditions and market clearing conditions.
3. Verify the conjectures.

Step 1

Conjecture B.1

$$\frac{S_t}{\mathcal{E}_t} = \frac{1}{\rho}. \tag{40}$$

Without loss of generality, we focus the optimists' optimization problem. Two state variables drive their portfolio allocation and consumption decisions: their wealth W^o and the relative wealth share $\omega = \frac{W^o}{W^o+W^p}$. Denote their value function as $\mathcal{J}(t, W^o, \omega)$. We also conjecture that

Conjecture B.2

$$\mathcal{J} = e^{-\rho t}\left[\frac{\log(W^o)}{\rho} + \mathcal{G}(\omega)\right] \tag{41}$$

where $\mathcal{G}(\omega)$ is a function of ω, and can be determined from HJB.

Substituting (41) into Bellman equation:

$$\begin{aligned} 0 = \sup_{\theta^o,\theta^{d,o},c^o} \Bigg\{ & -\log(W^o) + \frac{\theta^o\left(\mu^o + \rho - \lambda\mathbb{E}[k]\right) + \theta^{d,o} r^d + \left(1 - \theta^o - \theta^{d,o}\right) r^f - c^o}{\rho} - \frac{(\theta^o\sigma)^2}{2\rho} \\ & + \lambda\mathbb{E}\left[\frac{\log\left(1 + \theta^o k + \theta^{d,o} k^d\right)}{\rho}\right] + \log(c^o W^o) + \ldots \Bigg\} \end{aligned} \tag{42}$$

where ... denotes the terms involving ω and $\mathcal{G}(\omega)$. First-order conditions yield

$$\mu^o + \rho - \lambda\mathbb{E}[k] - r^f - \theta^o\sigma^2 + \lambda\mathbb{E}\left[\frac{k}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{43}$$

$$r^d - r^f + \lambda\mathbb{E}\left[\frac{k^d}{1 + \theta^o k + \theta^{d,o} k^d}\right] = 0 \tag{44}$$

$$c^o - \rho = 0 \tag{45}$$

Similarly, we can write down the Bellman equation and first order conditions for the pessimists.

Step 2

We start from (44). Note that unlike (43), (44) does not involve any belief heterogeneity.

Thus in equilibrium, when $k^d \neq 0$, it must be that

$$\theta^o + \eta\theta^{d,o} = 1 \tag{46}$$

$$\theta^p + \eta\theta^{d,p} = 1 \tag{47}$$

(46) and (47) satisfy market clearing conditions and investors' budget constraint, i.e., clearing of one market implies clearing of other markets. Note that (46) and (47) imply that investors' positions on risk-free debt are $(\eta-1)\theta^{d,o}$ and $(\eta-1)\theta^{p,o}$, respectively. Suppose risk-free debt is clear, then we have

$$0 = (\eta - 1)\left(\theta^{d,o}W^o + \theta^{d,p}W^p\right) \tag{48}$$

$$S = \theta^o W^o + \theta^p W^p \tag{49}$$

(48) and (49) imply the risky-debt market clearing and stock market clearing, respectively. Therefore,

$$\begin{aligned} r^d - r^f &= -\eta\lambda\mathbb{E}\left[\frac{k}{1+(\theta^o+\eta\theta^{d,o})k}\middle| k \leq \bar{k}\right]\mathbb{P}\left(k \leq \bar{k}\right) \\ &= -\eta\lambda\left(\frac{4}{(\bar{k}+2)^3} - \frac{3}{(\bar{k}+2)^2} - 1\right) \end{aligned} \tag{50}$$

We then move to (43) and solve θ^o and θ^p. With conditions (43) and (44) of both optimists and pessimists, we have

$$\mu^o - \mu^p = (\theta^o - \theta^p)\sigma^2 - \lambda\left(\int_{\bar{k}}^{\infty}\left(\frac{1}{1+\theta^o k} - \frac{1}{1+\theta^p k}\right)k p_K(k)\,dk\right) \tag{51}$$

where $p_K(k)$ is the p.d.f of $k = e^Y - 1$ and follows

$$p_K(k) = \frac{1}{\mathscr{B}(2,2)}\frac{k+1}{(k+2)^4} \tag{52}$$

Note that $\theta^p = \frac{1-\theta^o\omega}{1-\omega}$ by market clearing conditions. Also note that in equilibrium the threshold $\bar{k}$ must satisfy $\theta^o\bar{k} = \gamma$. Hence

$$\mathcal{Q}(\theta^o;\omega) := (\theta^o - \theta^p)\sigma^2 - \lambda\left(\int_{\bar{k}}^{\infty}\left(\frac{1}{1+\theta^o k} - \frac{1}{1+\theta^p k}\right)kp_K(k)\,dk\right) \tag{53}$$

is a function of θ^o.

$\mathcal{Q}(\theta^o;\omega)$ can be written as an analytical form, but θ^o does not have an analytical solution. We can further prove that $\mathcal{Q}(\theta^o;\omega)$ is a continuous function defined on $[1,\frac{1}{\omega}]$. By extreme value theorem, the $\max\limits_{[1,\frac{1}{\omega}]}\mathcal{Q}(\theta^o;\omega)$ is well defined.

When $\max\limits_{[1,\frac{1}{\omega}]}\mathcal{Q}(\theta^o;\omega) > \mu^o - \mu^p$, all other results remain except that θ^o cannot be solved through (51) any more. Since the optimists have exhausted all the shares they could possibly obtain from the market, $\theta^p = 0$. From market clearing conditions:

$$\theta^o\omega = 1 \Rightarrow \theta^o = \frac{1}{\omega} \tag{54}$$

Therefore, $\bar{k} = \gamma\omega$ and the credit spread follows (50).

Once θ^o is determined, substitute θ^o into (46) solves $\theta^{d,o}$. Substitute θ^o and $\theta^{d,o}$ into (43) results in r^f.

Step 3

The remaining step is to solve the stock price and verify Conjecture B.1. Note that

$$c^o = c^p = \rho \tag{55}$$

Therefore,

$$\mathscr{E} = \rho(W^o + W^p) = \rho S \tag{56}$$

The last equality comes from the set of market clearing conditions. $\square$

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Tables

Table 1: Baseline Parameters Values

Economy Fundamentals		
Long-run average growth of the aggregate endowment	μ	1.55%
Volatility of aggregate endowment growth	σ	3.44%
Volatility of the expected endowment growth	σ_μ	1.1%
Mean reversion of the expected endowment growth	α_μ	0.05
Default Write-down	η	0.694
Default Trigger	γ	-0.95
Belief Formation		
Behavior Bias	ϕ	0.61
Volatility of the signal(s)	σ_s	2.05
Time preference	ρ	0.03
Disaster Risk		
Long-run annual probability of disaster	$\bar{\lambda}$	1.169%
Volatility of disaster risk	σ_λ	0.082
Mean reversion of disaster risk	α_λ	0.11

The table shows parameter values in the calibration. Parameter values are expressed in annual terms.

Table 2: Moments On the Debt Markets: Model Generated Data and Historical Times Series

		No-jump simulations			All simulations		
	Data	5	50	95	5	50	95
r^f	2.69	-1.06	2.38	5.84	-1.05	2.38	5.85
$\sigma(r^f)$	2.21	1.36	2.13	3.45	1.34	2.12	3.42
$r^d - r^f$	99.99	79.09	91.25	104.90	78.59	90.87	104.21
$\sigma(r^d - r^f)$	67.87	49.45	66.16	97.54	49.26	66.17	96.66

The model is simulated at a monthly frequency and simulated data are aggregated to an annual frequency. Data moments on risk-free debt are calculated using 3-month treasury bill rate, from 1947 through the end of 2019. Data moments on risky debt are calculated using BofA AA US Corporate Index Option-Adjusted Spread, from 1997 to the end of 2019, the longest available series. The moments on risk-free debt are in percentage terms and the moments on risky debt are in basis point terms. The no-jump simulation column reports the 5th, 50th and 95th percentile for each statistic for the subset of simulations in which no rare events occur. The all Simulation column reports the 5th, 50th and 95th percentile for each statistic from all simulations.

Table 3: The Disruption of Disasters to the Credit Markets

$\mu^o - \mu^p$	ω	λ	$\bar{k}$	θ^o	L.G.D	Post Disaster ω
1.11%	50%	1.17%	-49.04%	1.94	23.43%	49.79%
1.50%	50%	1.17%	-47.58%	2.00	24.91%	49.74%
0.65%	50%	1.17%	-82.44%	1.15	0.00%	36.78%
1.11%	70%	1.17%	-70.08%	1.36	0.00%	24.62%
1.11%	30%	1.17%	-29.27%	3.25	33.68%	29.76%
1.11%	50%	1.29%	-49.81%	1.91	22.68%	49.81%
1.11%	50%	1.08%	-48.46%	1.96	24.01%	49.78%

This table shows wealth redistribution and loss given default as a fraction of total wealth for different states of the economy when the endowment suddenly drops by 50%. The first line features baseline parameter values.

Table 4: Risky Debt and Risk-Free Debt

	Total Risky	Total Risky	Total Risky	Non Fin Bus & Household	Non Fin Corp Bus
	(1)	(2)	(3)	(4)	(5)
Federal Debt	5.787***	5.367***	6.241***	2.777***	0.126*
	(1.368)	(1.352)	(1.917)	(0.554)	(0.068)
Federal Debt (t-1)	−0.377	−0.515	0.313	−0.434	−0.039
	(1.386)	(1.351)	(1.693)	(0.548)	(0.083)
Federal Debt (t-2)	−3.481***	−3.352***	−3.341*	−1.647***	0.106
	(1.235)	(1.231)	(1.935)	(0.498)	(0.070)
1-yr Treasury Rate		−3.322	4.136*	−0.985	−0.246*
		(2.382)	(2.439)	(0.956)	(0.144)
10-yr Treasury Rate		−0.472	−6.458**	−0.255	0.109
		(2.816)	(2.876)	(1.150)	(0.158)
Constant	0.740***	1.192***	0.369**	0.866***	0.106***
	(0.044)	(0.172)	(0.152)	(0.069)	(0.011)
Observations	214	214	168	214	214
R^2	0.668	0.685	0.696	0.713	0.859

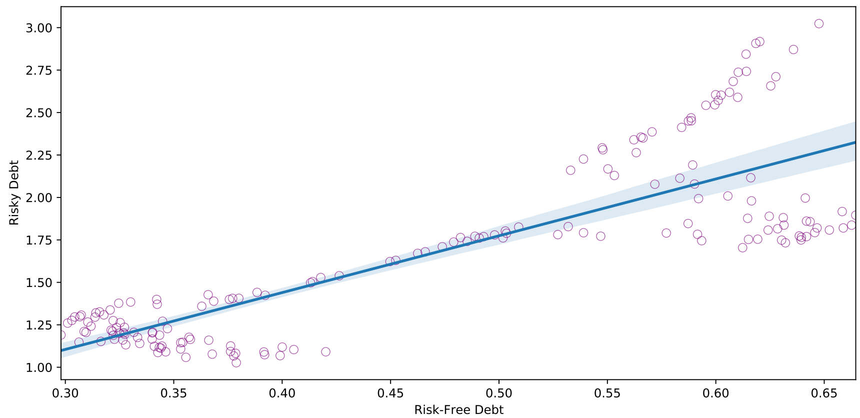
Note: This table shows the regression results of risky debt on federal debt. The total risky debt is measured as total debt outstanding minus federal debt normalized by GDP. Nonfinancial business and household debt is measured as the sum of debt and loans issued by nonfinancial business, households and nonprofit organizations normalized by GDP. Nonfinancial corporate business debt is measured as debt securities issued by nonfinancial corporate business normalized by GDP. All columns use the quarterly data from 1966 to 2019 except column (3), which uses data up to the first quarter of 2008. Heteroskedasticity and autocorrelation consistent standard errors are reported in the parentheses. ***, **, * denote significance at the 1%, 5%, and 10% levels, respectively.

Table 5: The Conditional Distribution of Optimists' Survival

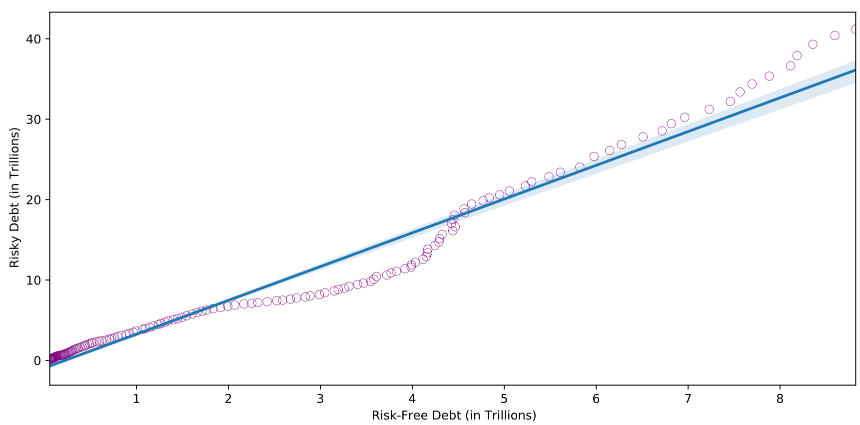
Initial ω	Mean	5%	50%	95%
50%	12.02%	1.11%	10.83%	27.63%
10%	0.08%	$< 0.01\%$	$< 0.01\%$	0.28%
85%	68.47%	46.08%	69.49%	87.47%

The table shows the optimists' wealth share in 100 years for different initial wealth distributions. For each initial wealth distribution, we simulate 10,000 paths and reports the mean and 5th, 50th and 95th percentiles of the final wealth share for the subset of simulations in which no rare disasters occur.

Figures



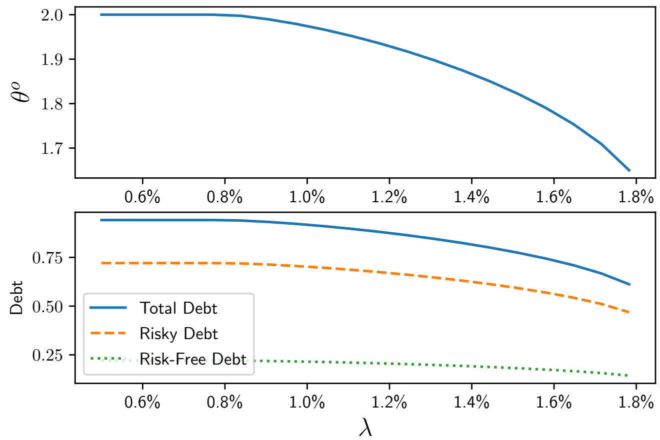
(A)



(B)

Figure 1: **Risky debt and risk-free debt**

The figure plots quarterly amount of US federal debt and other types of debt securities from 1966 to 2008. The straight line is the fitted regression line. The data of federal debt is from US Department of the Treasury. The data of total amount of debt is from the Federal Reserve. The amount of risky debt is measured as their difference. Panel A and B normalize the amount of debt by GDP and GDP deflator, respectively.



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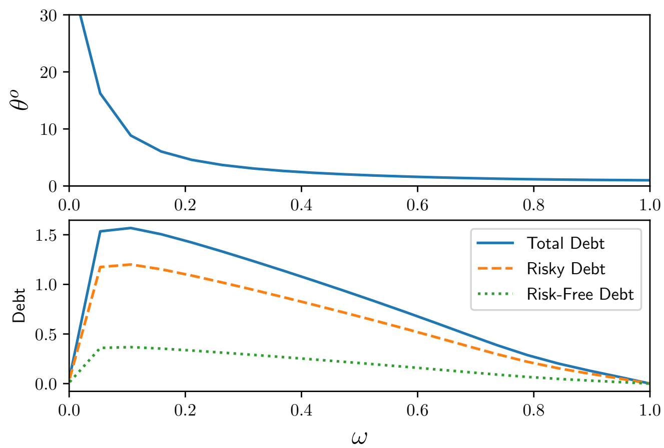

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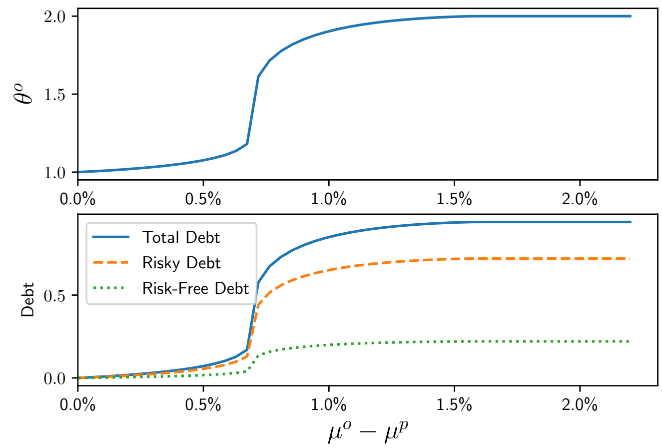


(C)

Figure 2: **States of the economy, leverage and total debt**

Panel A, B and C plot the effects of disaster intensity, wealth distribution and belief dispersion on optimists' investment share θ^o on stocks and all types of debt as a fraction of the total wealth in the economy, respectively.

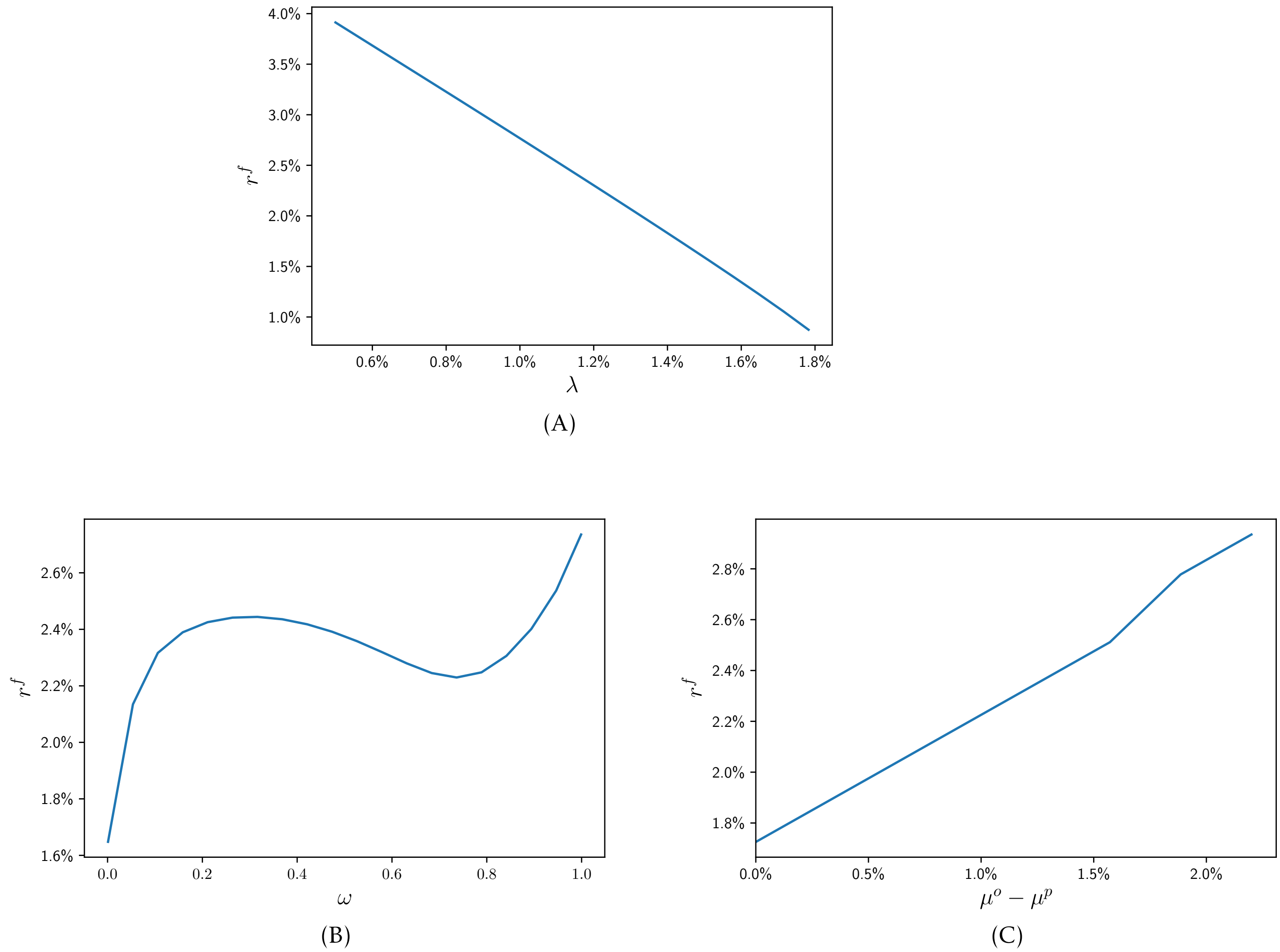
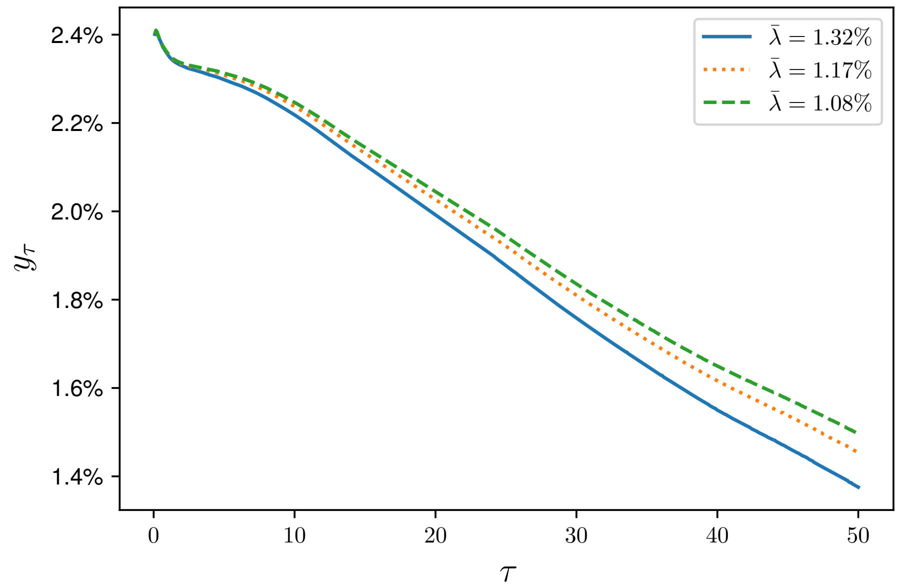
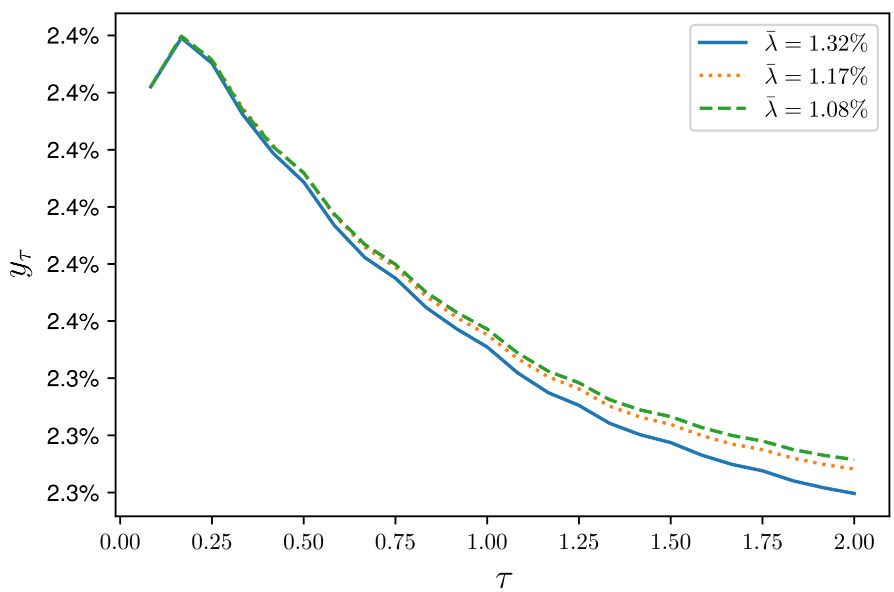


Figure 3: **Risk-free rate**

Pane A plots the risk-free rate as a function of disaster intensity. Panel B and C plot the risk-free rate as a function of wealth distribution and belief dispersion, respectively.



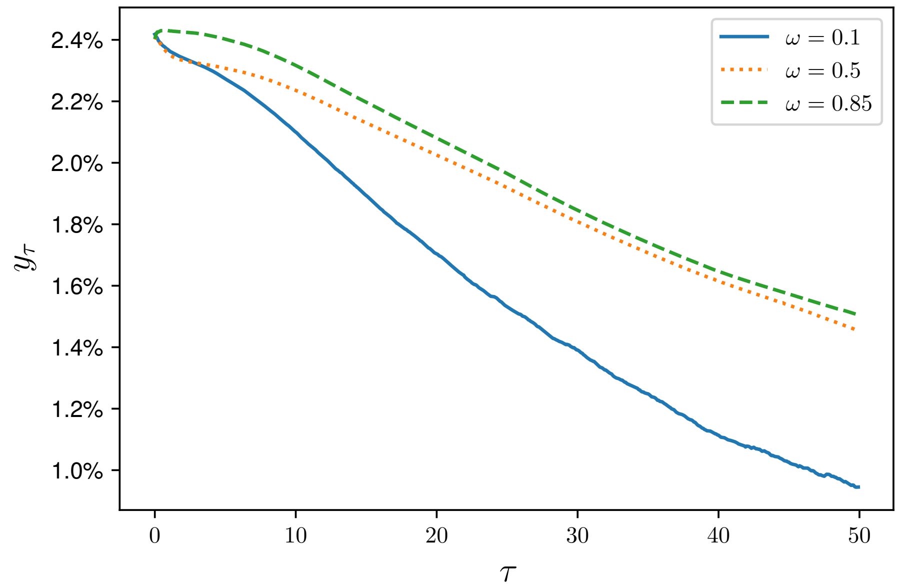
(A)



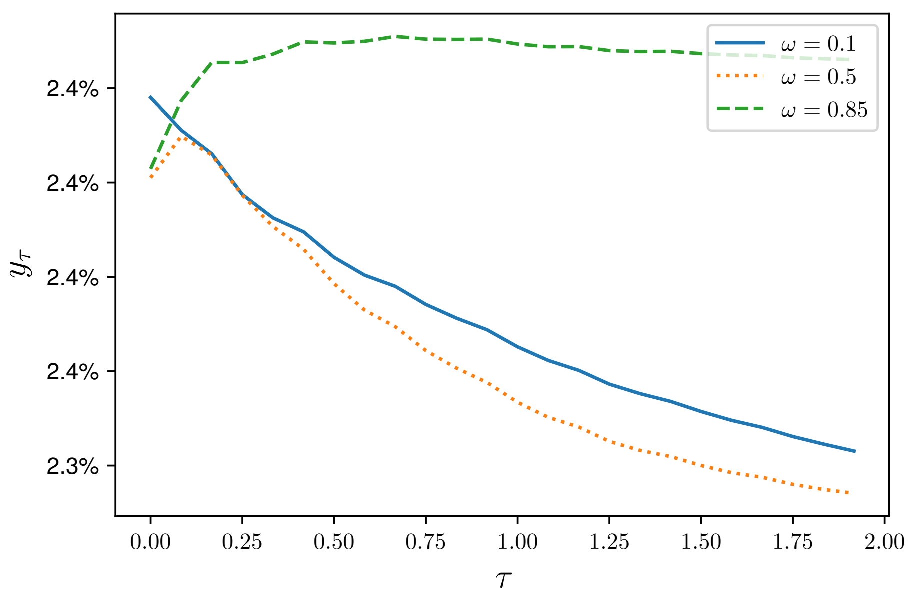
(B)

Figure 4: **Risk-free Yield Curve and disaster risk**

Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\bar{\lambda} = 1.32\%$. The dotted and dashed lines correspond to $\bar{\lambda} = 1.17\%$ and $\bar{\lambda} = 1.08\%$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.



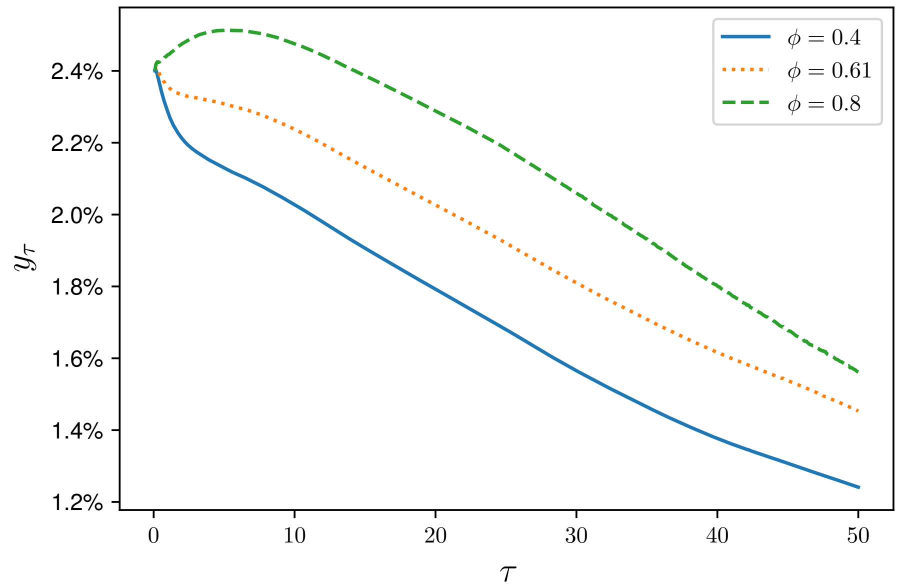
(A)



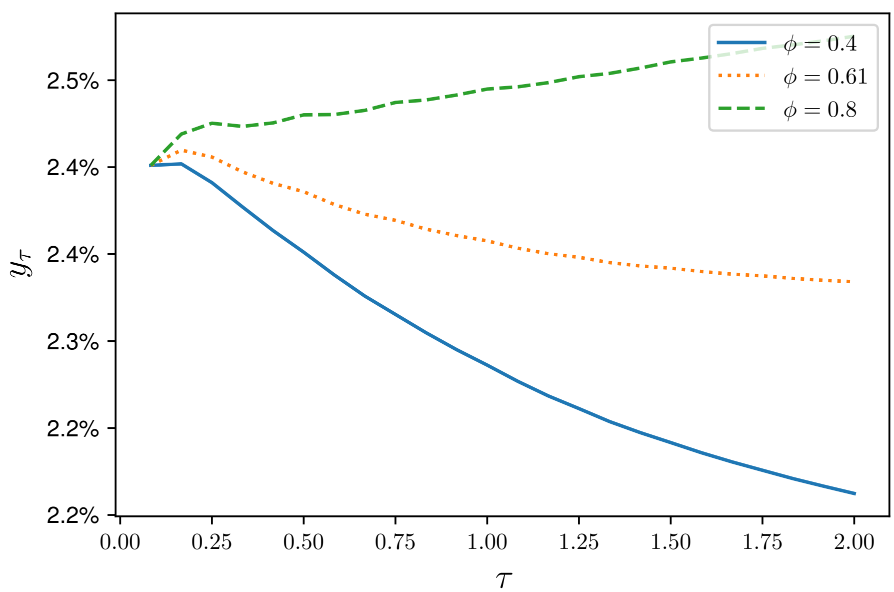
(B)

Figure 5: **Risk-free Yield Curve and Wealth Distribution**

Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\omega = 0.1$. The dotted and dashed lines correspond to $\omega = 0.5$ and $\omega = 0.85$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.



(A)



(B)

Figure 6: **Risk-free Yield Curve and Belief Dispersion**

Panel A plots the yield curves for maturities from 0 to 50 years. The solid line is for the case of $\phi = 0.4$. The dotted and dashed lines correspond to $\phi = 0.61$ and $\phi = 0.8$, respectively. Panel B zooms in the curves for maturities ranging from 0 to 2 years.

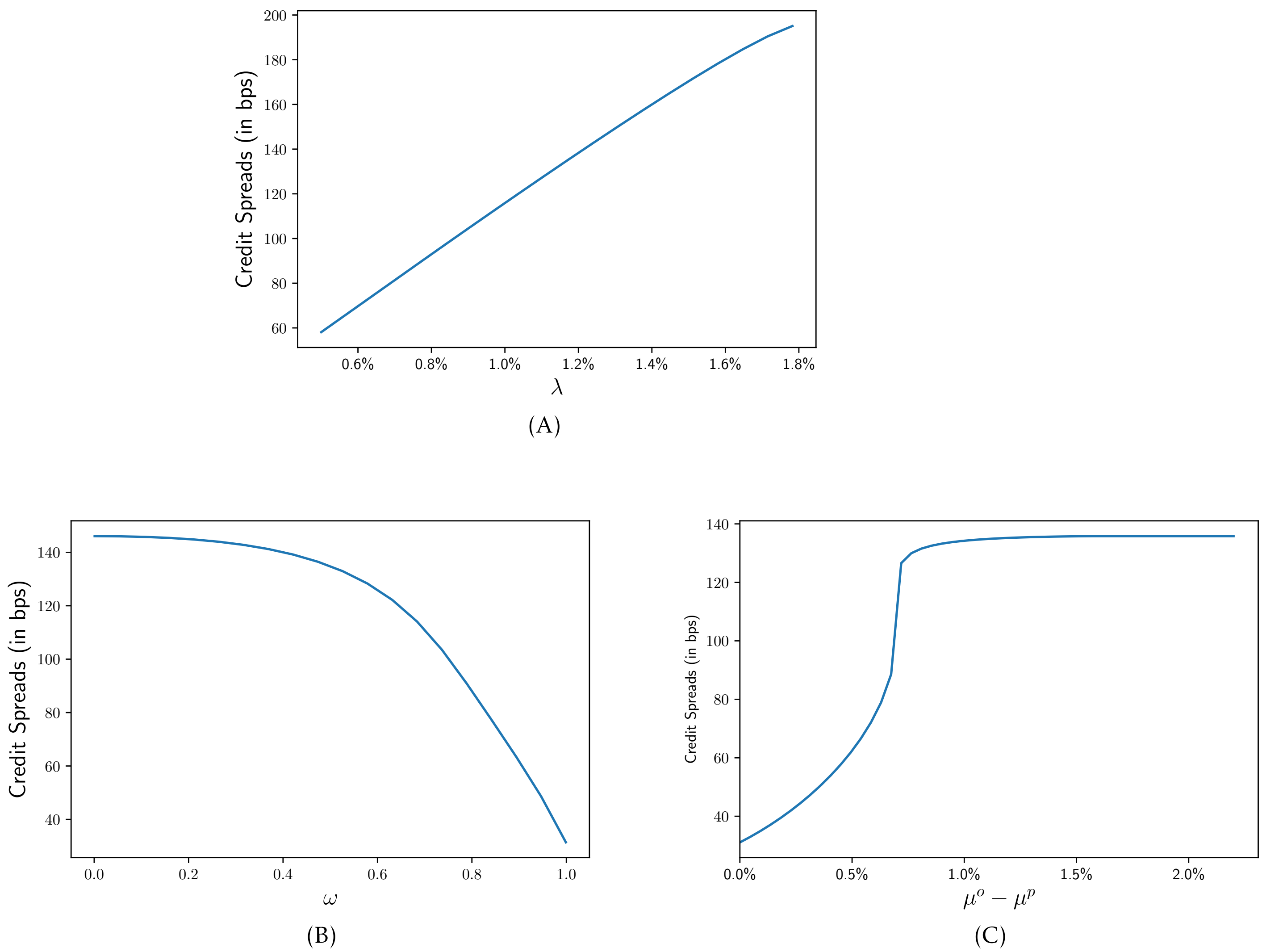
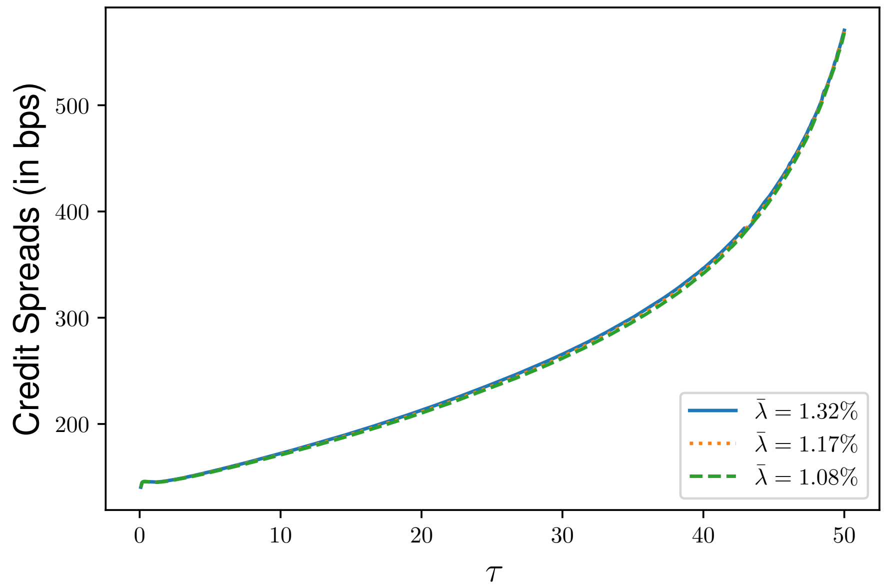
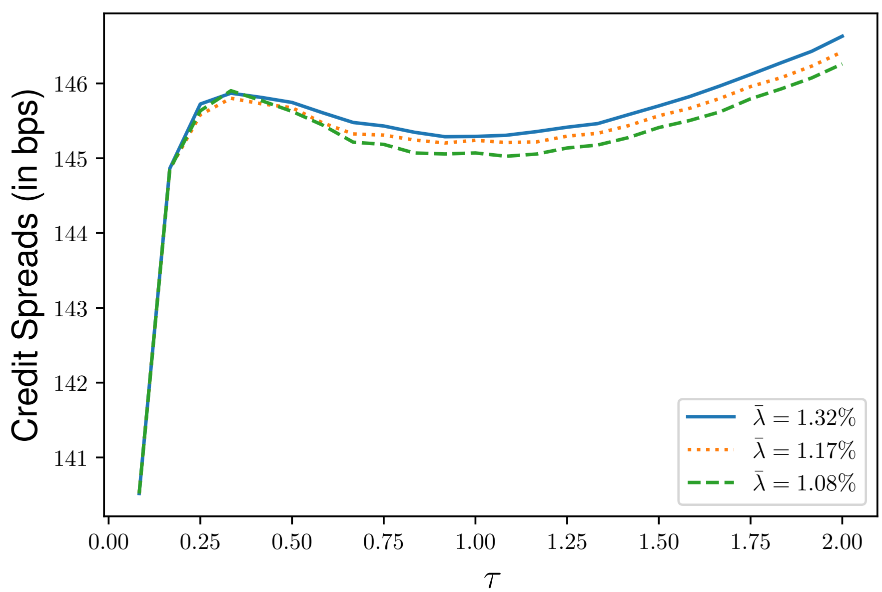


Figure 7: **credit spreads and the states of the economy**

Panel A plots the effect of disaster intensity on instantaneous credit spreads. Panel B and C plot the effect of wealth distribution and belief dispersion on credit spreads, respectively.



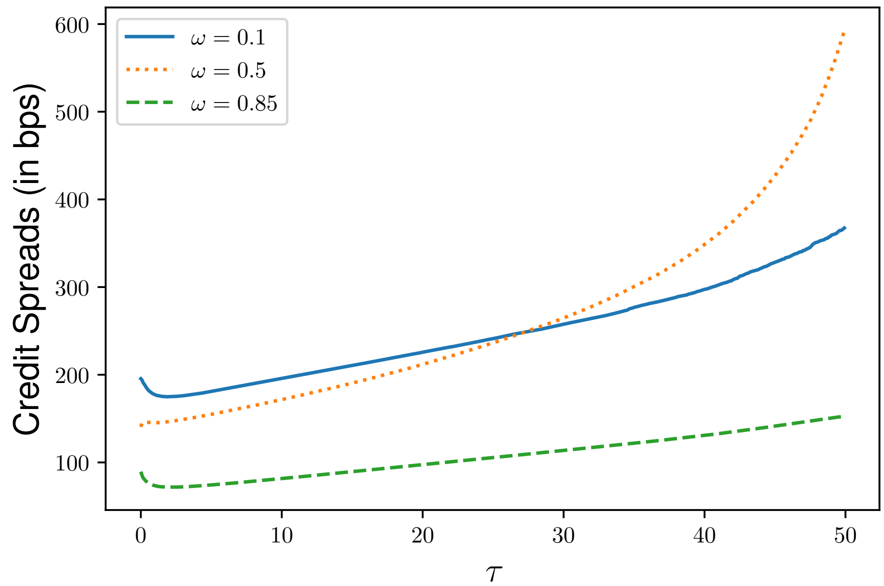
(A)



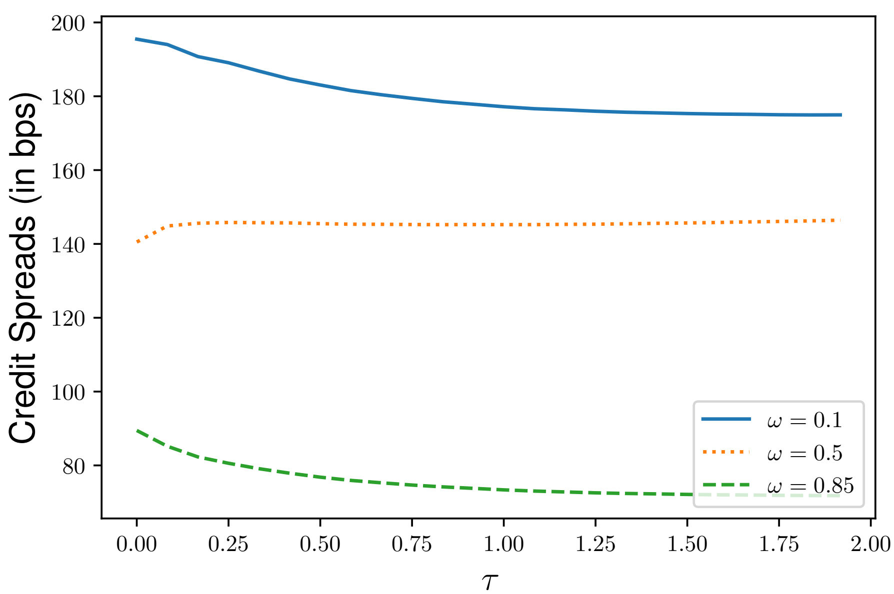
(B)

Figure 8: **Term structure of credit spreads and disaster risk**

Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\bar{\lambda} = 1.32\%$. The dotted and dashed lines correspond to $\bar{\lambda} = 1.17\%$ and $\bar{\lambda} = 1.08\%$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.



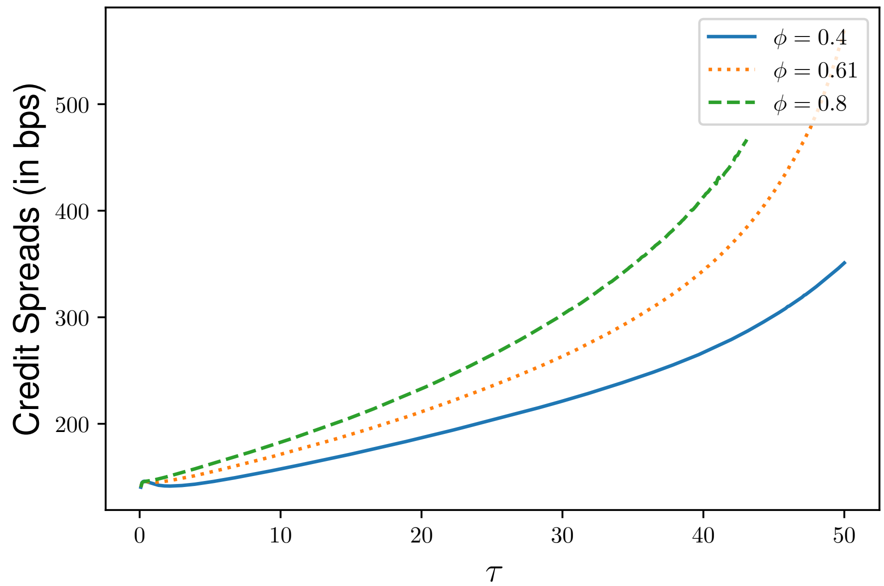
(A)



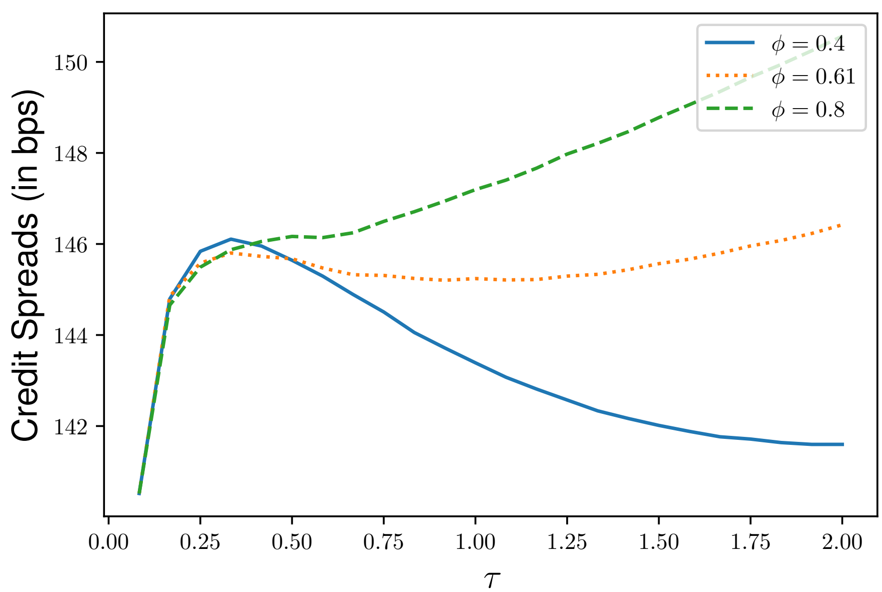
(B)

Figure 9: **Term structure of credit spreads and wealth distribution**

Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\omega = 0.1$. The dotted and dashed lines correspond to $\omega = 0.5$ and $\omega = 0.85$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.



(A)



(B)

Figure 10: **Term structure of credit spreads and belief dispersion**

Panel A plots the term structure of credit spreads for maturities from 0 to 50 years. The solid line is for the case of $\phi = 0.4$. The dotted and dashed lines correspond to $\phi = 0.61$ and $\phi = 0.8$, respectively. Panel B zooms in the lines on maturities ranging from 0 to 2 years.

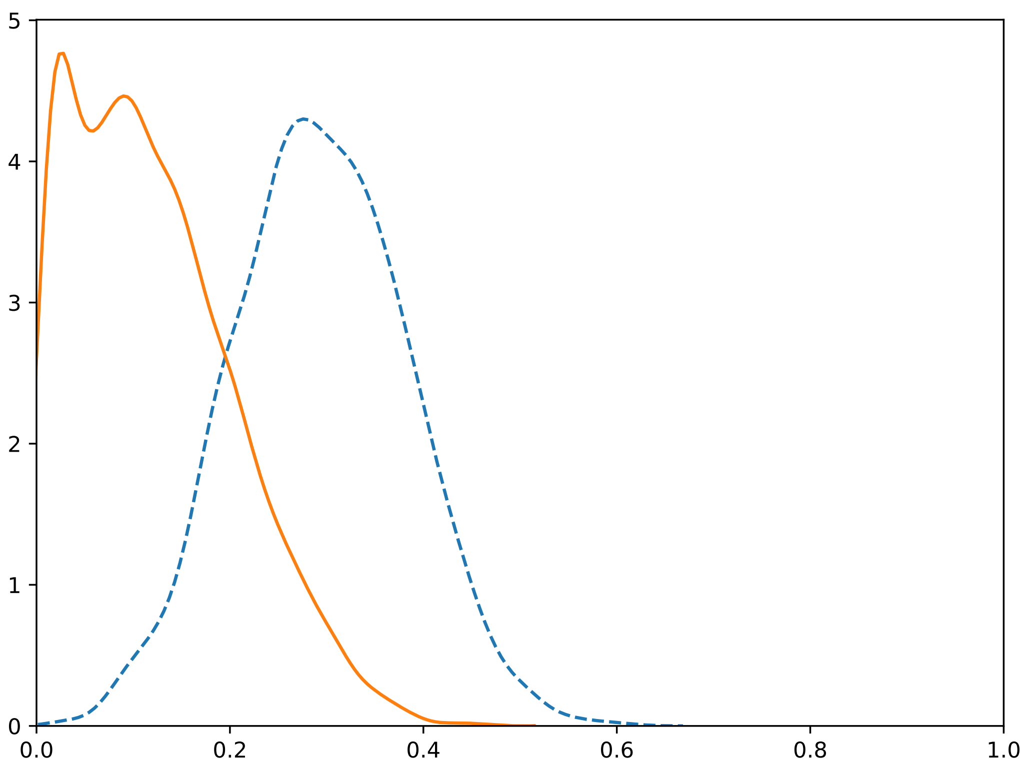
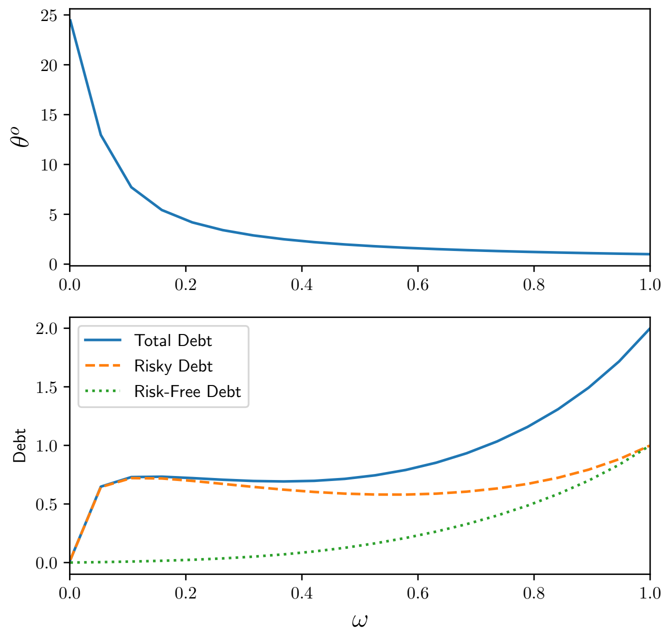
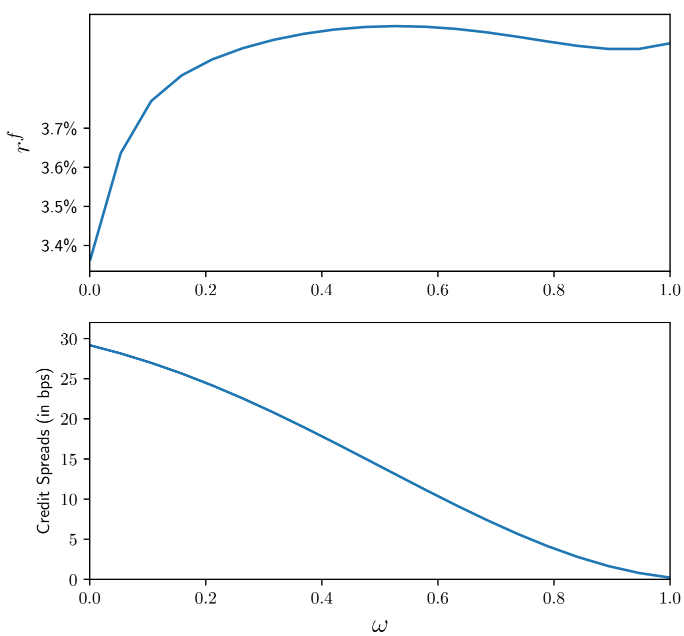


Figure 11: **Belief Dispersion and Survival**

The figure plots the distribution of the optimists' wealth share in 100 years, given the initial wealth share is 0.5. The solid line plots the final distribution when both types of investors have symmetric belief bias. The dashed line plots the wealth distribution when the optimists' belief bias is half of that of the pessimists' on average.



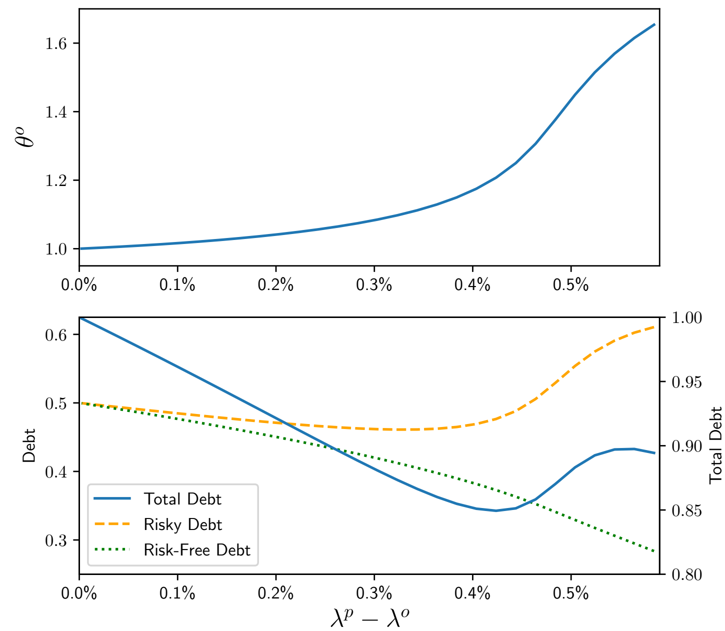
(A)



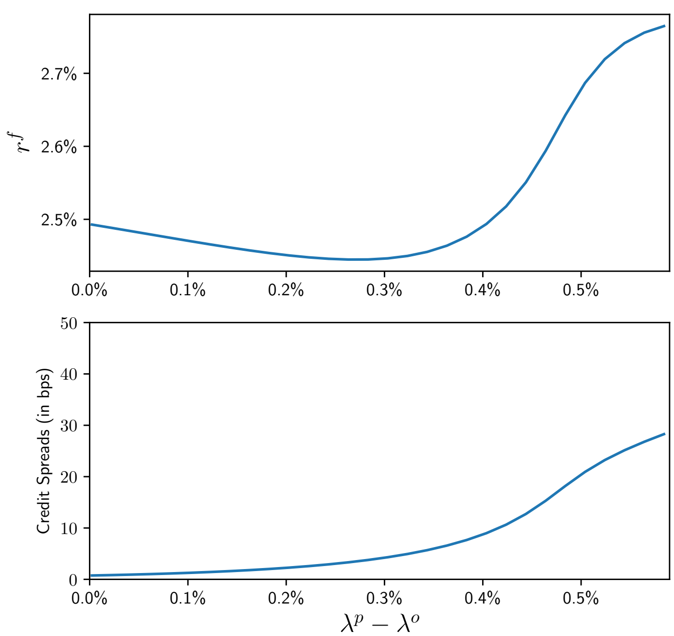
(B)

Figure 12: **Disagreement over Disaster Risk**

Panel A plots the effect of wealth distribution on optimists' investment share θ^o on stocks and total debt as a fraction of the total wealth in the economy. Panel B plots the effect of wealth distribution on instantaneous risk-free rate and credit spreads. $\lambda^p = 1.168\%$, $\lambda^o = \frac{\lambda^p}{2} = 0.584\%$. All other parameter values follow Table 1.



(A)



(B)

Figure 13: **Disagreement over Disaster Risk**

Panel A plots the effect of belief dispersion on optimists' investment share θ^o on stocks and total debt as a fraction of the total wealth in the economy. Panel B plots the effect of belief dispersion on instantaneous risk-free rate and credit spreads. We fix the pessimist's belief λ^p and adjust the optimist's belief λ^o. All other parameter values follow Table 1.

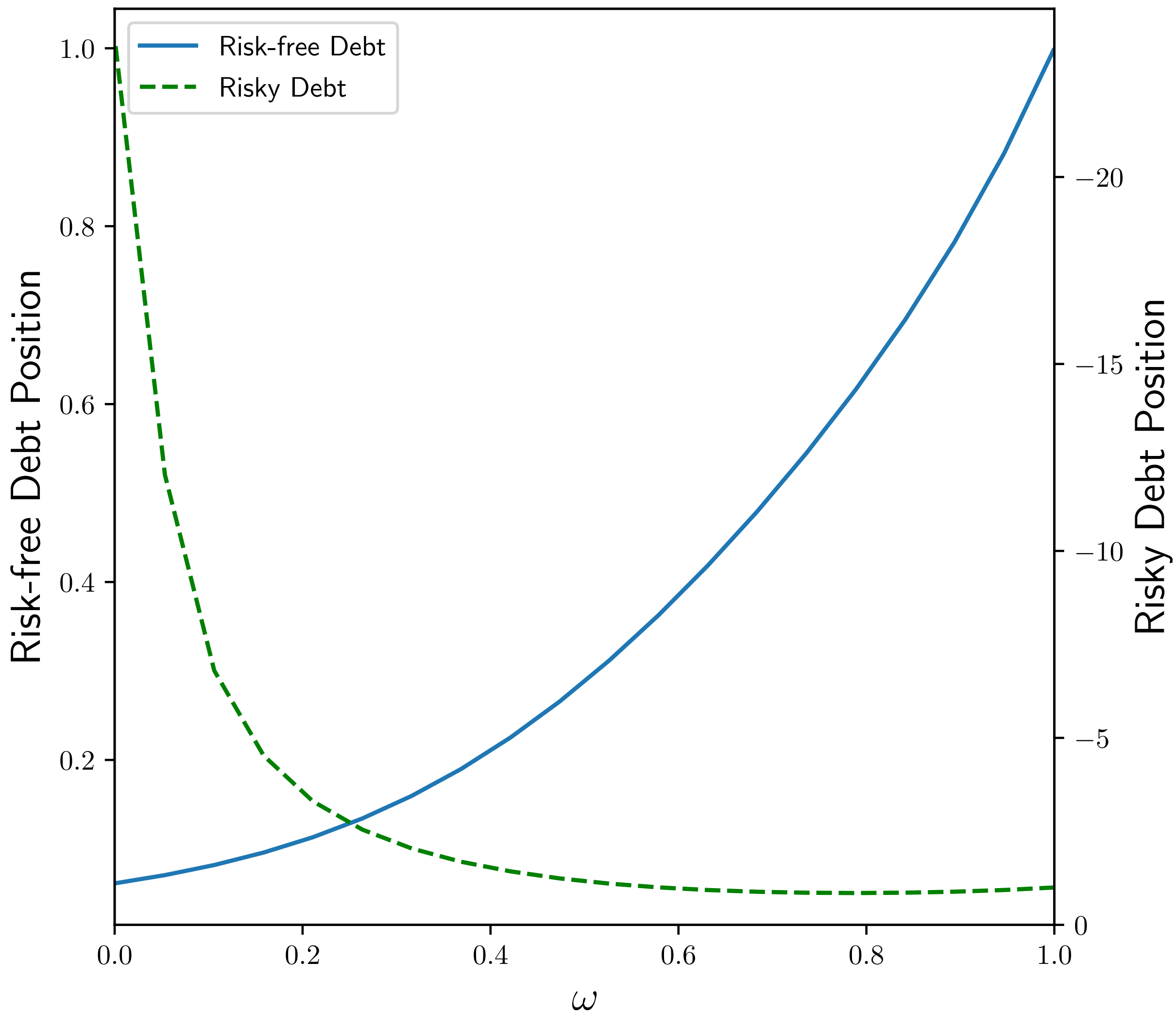


Figure 14: **Risky Debt, Risk-Free Debt and Disagreement over Disaster Risk**

The figure plots the optimists' risky debt and risk-free debt positions as functions of their wealth share ω. $\lambda^p = 1.168\%$, $\lambda^o = \frac{\lambda^p}{2} = 0.584\%$. All other parameter values follow Table 1.